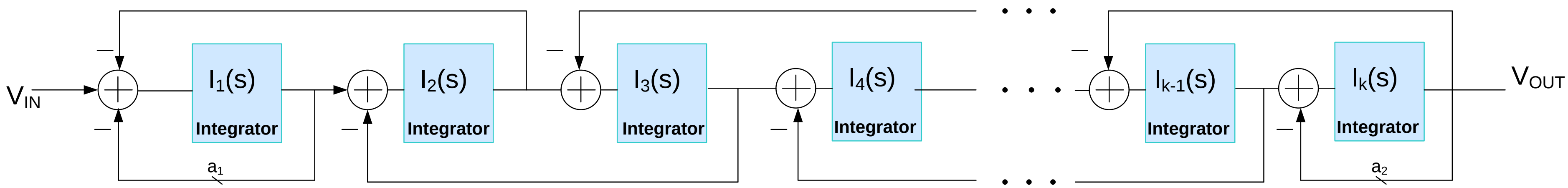
EE 508
Lecture 31

Transconductor Design

Review from last lecture

Leapfrog Filters

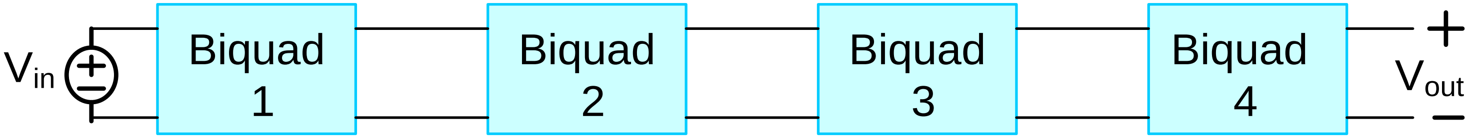


Introduced by Girling and Good, Wireless World, 1970

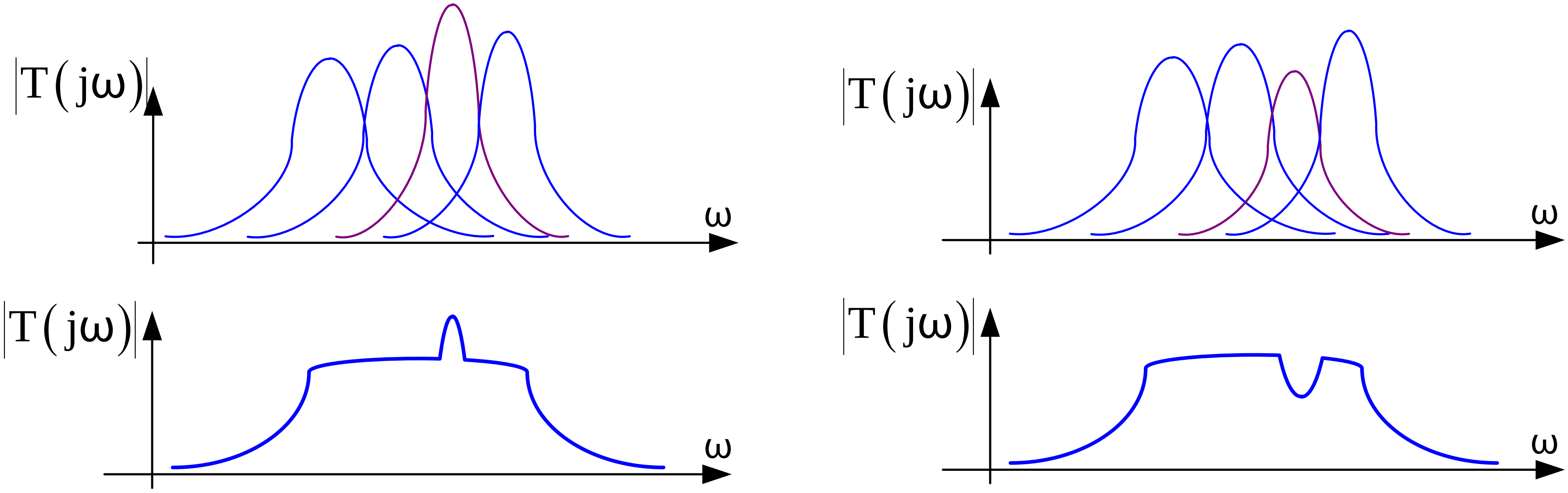
This structure has some very attractive properties and is widely used though the real benefits and limitations of the structure are often not articulated

Review from last lecture

Implications of Theorem 1



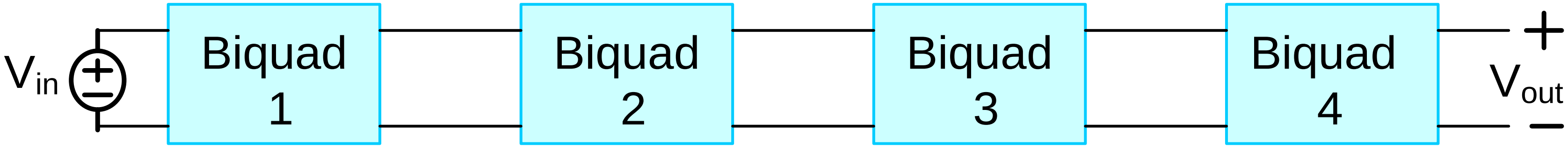
If a component in a biquad changes a little, there is often a large change in the passband gain characteristics (depicted as bandpass)

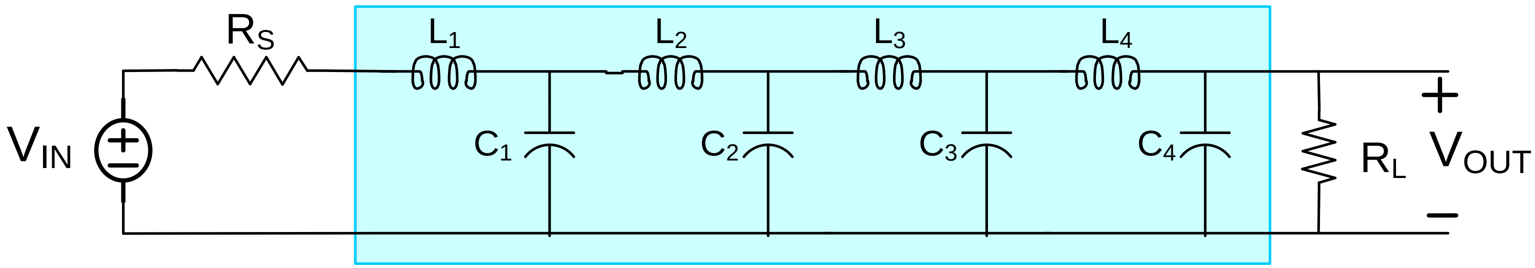


$$S_x^{|T(j\omega)|} \neq 0 \quad \text{in passband}$$

Review from last lecture

Implications of Theorem 1



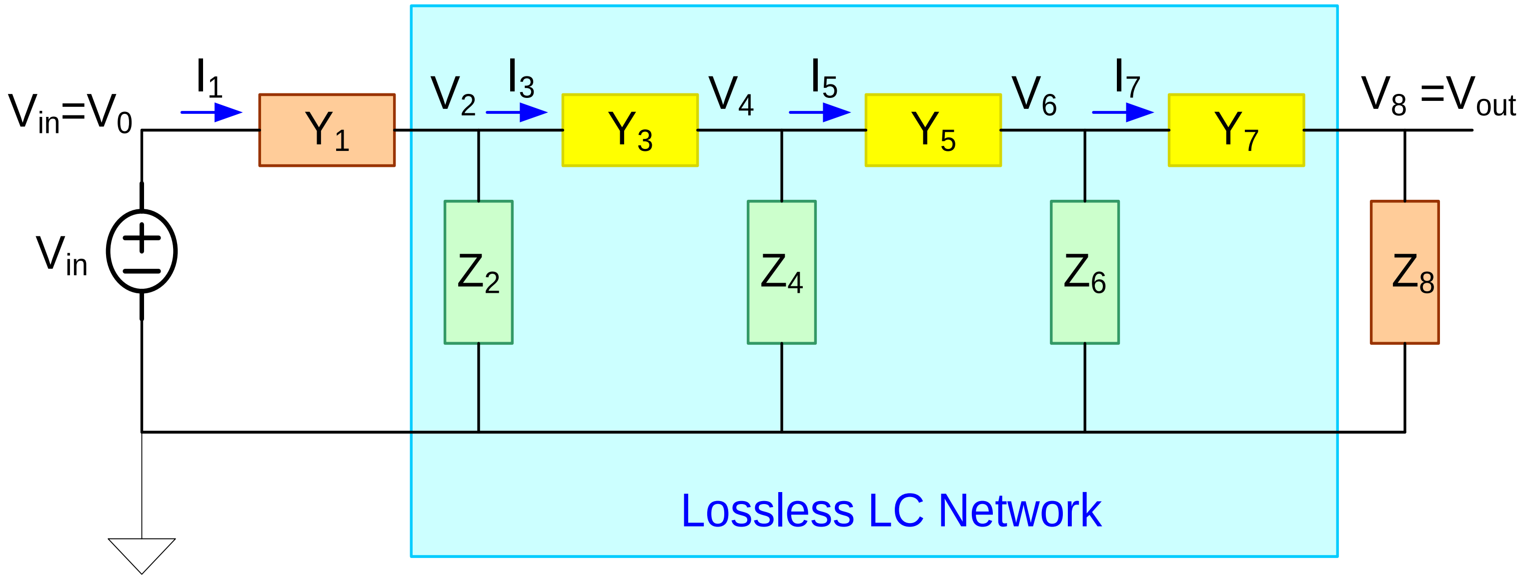


Good doubly-terminated LC networks often much less sensitive to most component values in the passband than are cascaded biquads !

This is a major advantage of the LC networks but can not be applied practically in most integrated applications or even in pc-board based designs

Review from last lecture

Doubly-terminated Ladder Network with Low Passband Sensitivities



$$
\left.
\begin{aligned}
I_1 &= (V_0 - V_2)Y_1 \\
V_2 &= (I_1 - I_3)Z_2 \\
I_3 &= (V_2 - V_4)Y_3 \\
V_4 &= (I_3 - I_5)Z_4 \\
I_5 &= (V_4 - V_6)Y_5 \\
V_6 &= (I_5 - I_7)Z_6 \\
I_7 &= (V_6 - V_8)Y_7 \\
V_8 &= I_7 Z_8
\end{aligned}
\right\}
$$

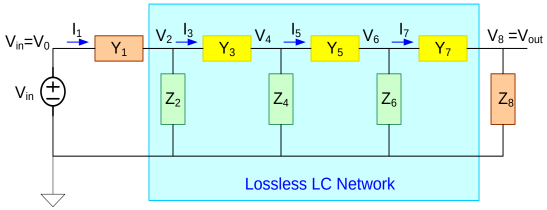
Complete set of independent equations that characterize this filter

Solution of this set of equations is tedious

All sensitivity properties of this circuit are inherently embedded in these equations!

Review from last lecture

Consider now only the set of equations and disassociate them from the circuit from where they came



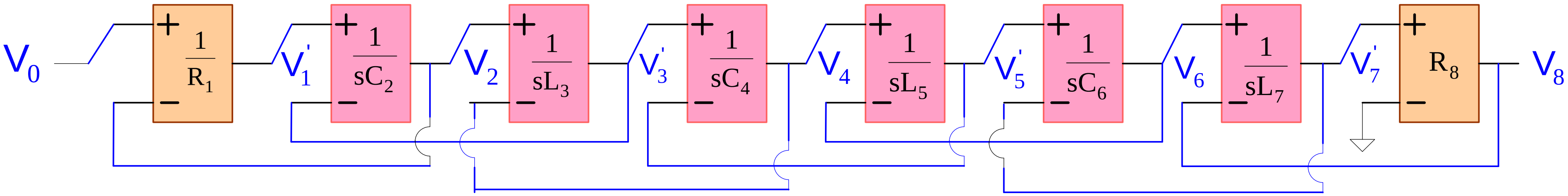
$$V_1' = (V_0 - V_2)\frac{1}{R_1} \qquad V_5' = (V_4 - V_6)\frac{1}{sL_5}$$

$$V_2 = (V_1' - V_3')\frac{1}{sC_2} \qquad V_6 = (V_5' - V_7')\frac{1}{sC_6}$$

$$V_3' = (V_2 - V_4)\frac{1}{sL_3} \qquad V_7' = (V_6 - V_8)\frac{1}{sL_7}$$

$$V_4 = (V_3' - V_5')\frac{1}{sC_4} \qquad V_8 = V_7' R_8$$

The interconnections that complete each equation can now be added



Review from last lecture

Bandpass Leapfrog Structures

Consider lowpass to bandpass transformations

Un-normalized

$$s_n \rightarrow \frac{s^2 + \omega_0^2}{sBW}$$

$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2 + \omega_0^2}$$

$$\frac{1}{s_n + \alpha} \rightarrow \frac{sBW}{s^2 + s\alpha BW + \omega_0^2}$$

Normalized

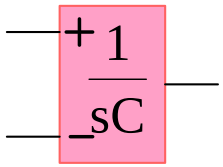
$$s_n \rightarrow \frac{s^2 + 1}{sBW_n}$$

$$\frac{1}{s_n} \rightarrow \frac{sBW_n}{s^2 + 1}$$

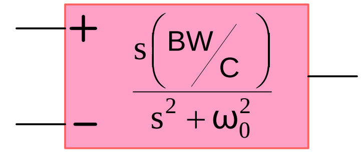
$$\frac{1}{s_n + \alpha} \rightarrow \frac{sBW_n}{s^2 + s\alpha BW_n + 1}$$

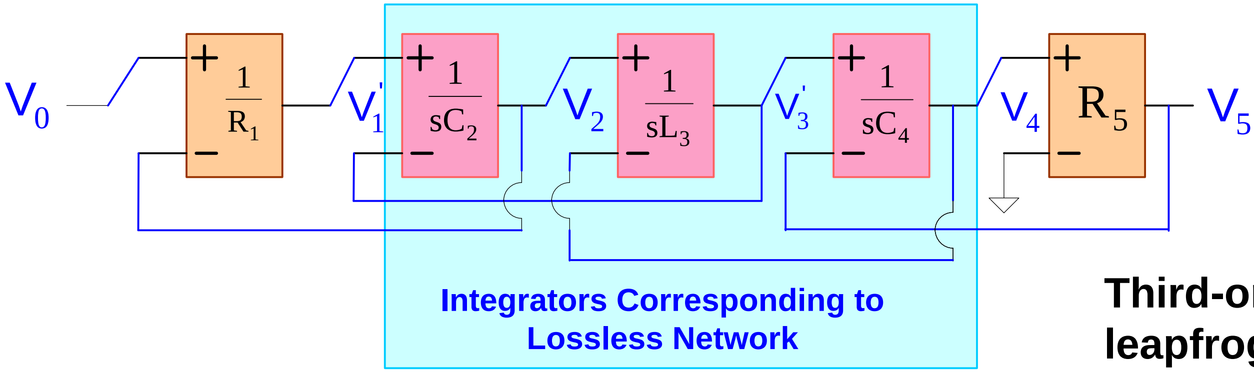
Review from last lecture

Bandpass Leapfrog Structures

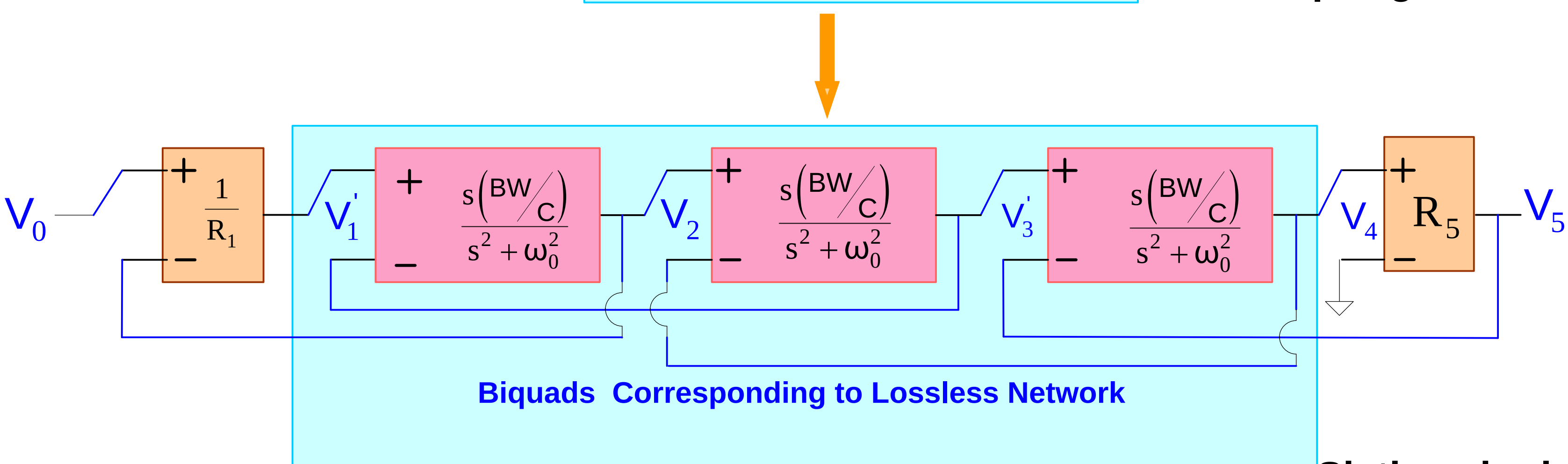


$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2+\omega_0^2}$$





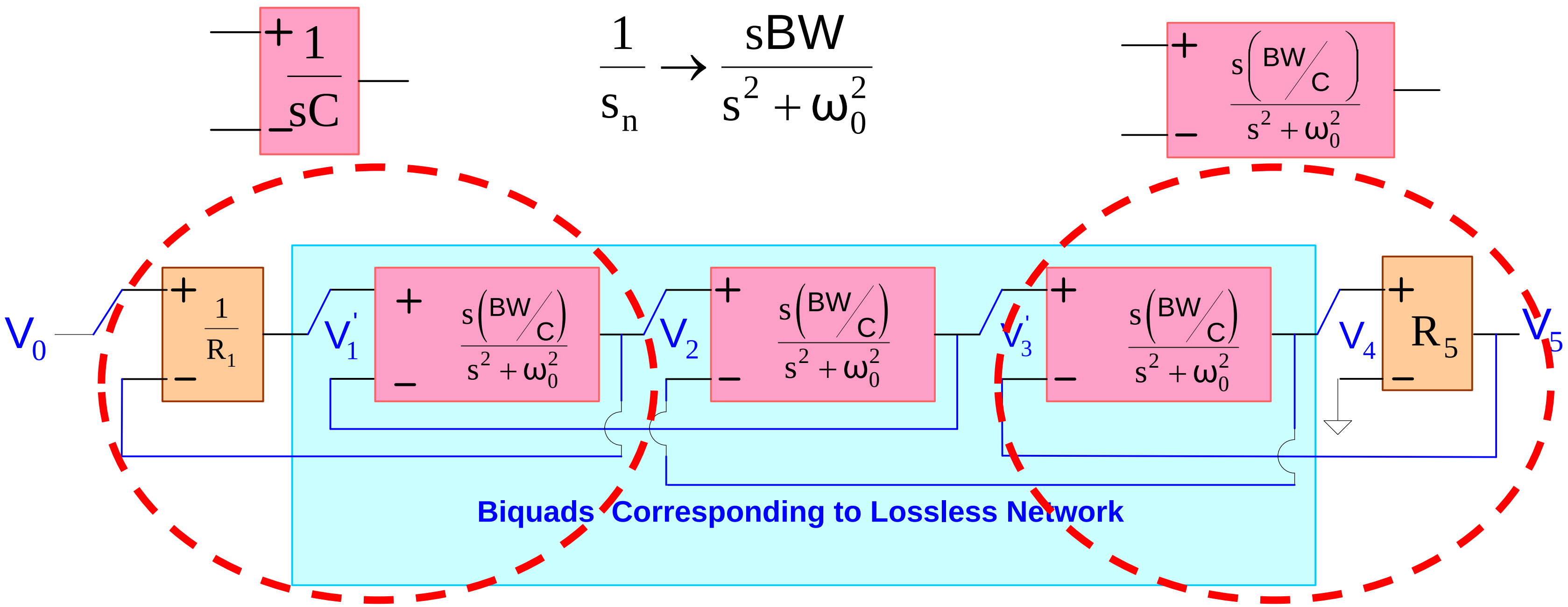
Third-order lowpass leapfrog filter



Sixth-order bandpass leapfrog filter

Review from last lecture

Bandpass Leapfrog Structures



"Loss" at input and/or output can usually be incorporated into finite-Q terminating biquads instead of requiring additional voltage amplifiers

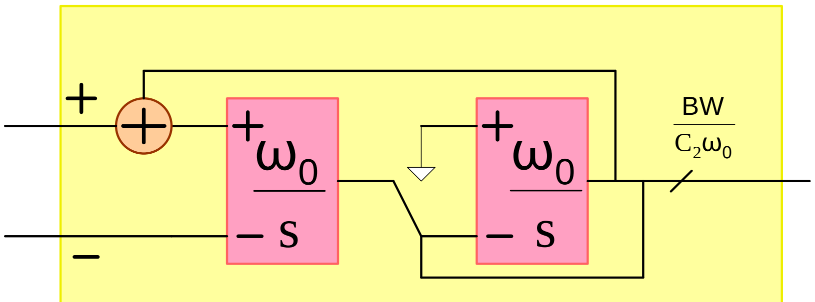
Review from last lecture

Bandpass Leapfrog Structures

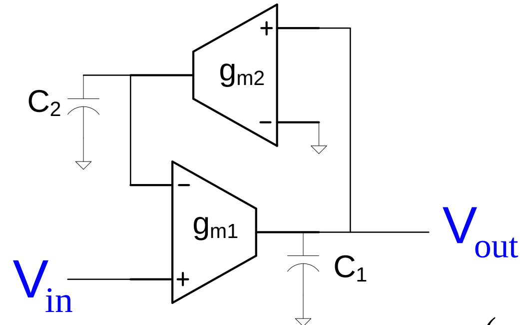
Integrator-based biquads

OTA-C Implementations (Concept)

Infinite Q bandpass biquad



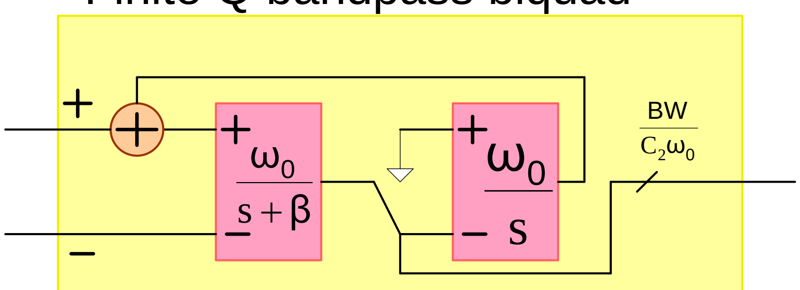
$$T(s)=\frac{s\left(BW/C\right)}{s^2+\omega_0^2}$$



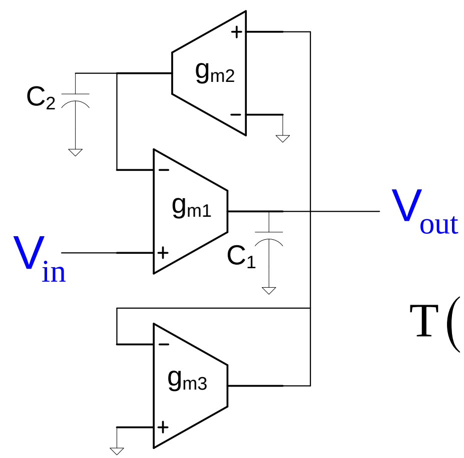
(Not Differential)

$$T(s)=\frac{s\left(\frac{g_{m1}}{C_1}\right)}{s^2+\frac{g_{m1}g_{m2}}{C_1C_2}}$$

Finite Q bandpass biquad



$$T(s)=\frac{s\left(BW/C\right)}{s^2+s\alpha BW+\omega_0^2}$$



(Not Differential)

$$T(s)=\frac{s\left(\frac{g_{m1}}{C_1}\right)}{s^2+s\frac{g_{m3}}{C_1}+\frac{g_{m1}g_{m2}}{C_1C_2}}$$

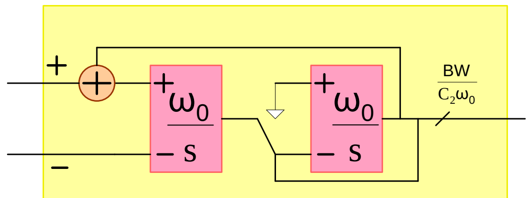
Review from last lecture

Bandpass Leapfrog Structures

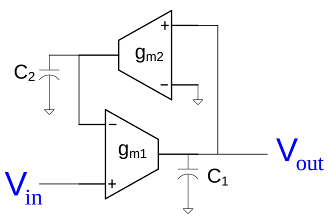
Integrator-based biquads

OTA-C Implementations

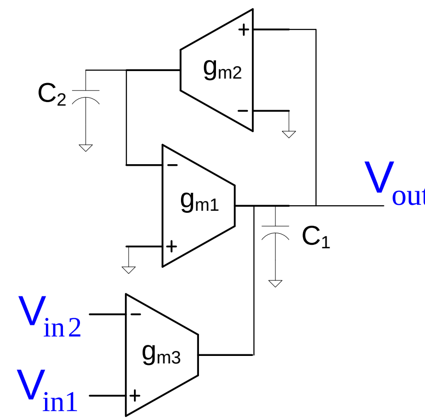
Infinite Q bandpass biquad



$$T(s)=\frac{s\left(BW/C\right)}{s^2+\omega_0^2}$$



$$T(s)=\frac{s\left(\frac{g_{m1}}{C_1}\right)}{s^2+\frac{g_{m1}g_{m2}}{C_1C_2}}$$

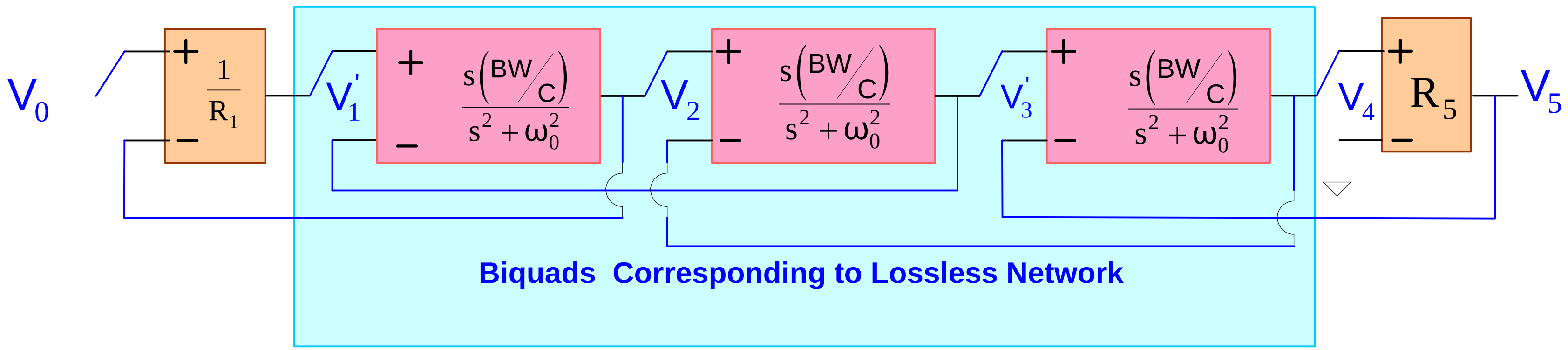


$$V_{OUT}(s)=\frac{s\left(\frac{g_{m3}}{C_1}\right)\left[V_{in1}-V_{in2}\right]}{s^2+\frac{g_{m1}g_{m2}}{C_1C_2}}$$

Multiple inputs can be added to lossy integrator too!

Review from last lecture

Bandpass Leapfrog Structures



Note the lossless biquads are infinite Q structures !

Is it easy or practical to implement infinite Q biquads?

Yes – have shown by example in g_m-C family and also easy in other families

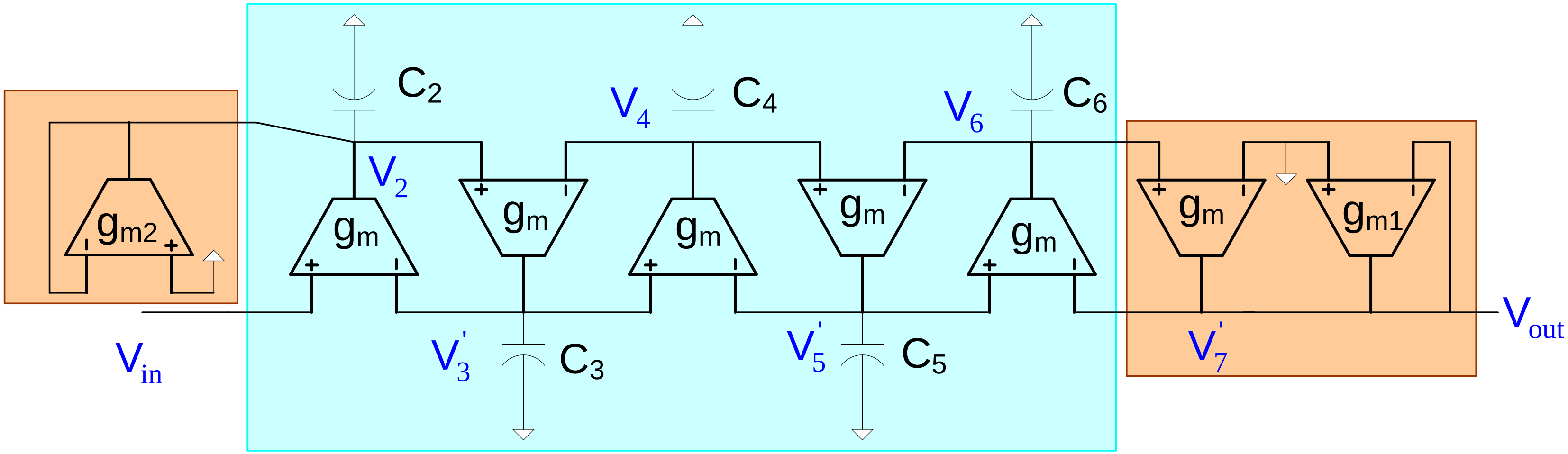
Are there stability concerns about the infinite Q biquads?

Stability of overall leapfrog structure of concern, not stability of individual biquads
Overall leapfrog structure is robust with low passband sensitivities !

Review from last lecture

Leapfrog Implementations

Fifth-order Lowpass Leapfrog with OTAs



$$V_1' = \frac{1}{R_1}\left(V_{in} - V_2\right)$$

$$V_2 = \frac{g_m/C_2}{s}\left(V_1' - V_3'\right)$$

$$V_3' = \frac{g_m/C_3}{s}\left(V_2 - V_4\right)$$

$$V_4 = \frac{g_m/C_4}{s}\left(V_3' - V_5'\right)$$

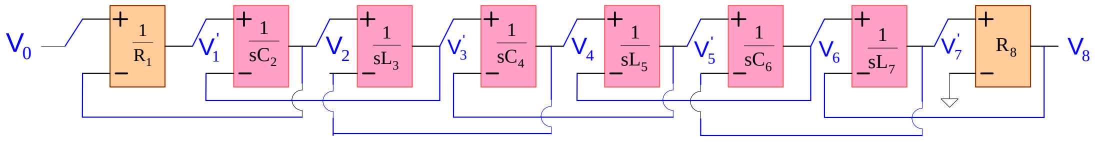
$$V_5' = \frac{g_m/C_5}{s}\left(V_4 - V_6\right)$$

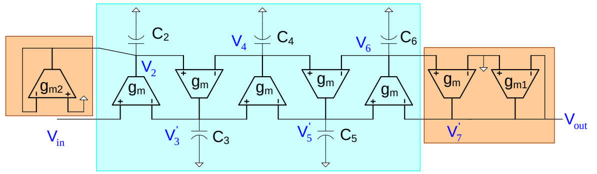
$$V_6 = \frac{g_m/C_6}{s}\left(V_5 - V_7'\right)$$

$$V_7' = \left(g_m/g_{m1}\right)V_6$$

Practically can either fix g_ms and vary capacitors or fix capacitors and vary g_m's

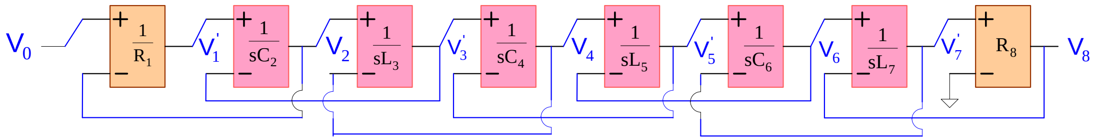
Leapfrog Filters
A Seminal Contribution

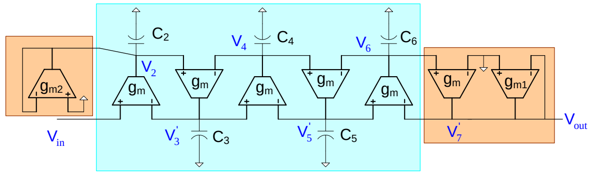




- A valuable contribution ?
- A timely contribution ?
- A clever idea?
- Would someone else have come up with it had Girling and Good not made the discovery?
- Example of unlikely publication making major disclosure

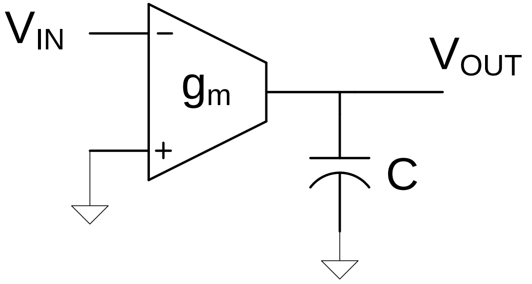
Leapfrog Filters
A Seminal Contribution





- A valuable contribution ?
- A timely contribution ?
- A clever idea?
- Would someone else have come up with it had Girling and Good not made the discovery?
- Example of unlikely publication making major disclosure

Transconductor Design



Transconductor-based filters depend directly on the g_m of the transconductor

Feedback is not used to make the filter performance insensitive to the transconductance gain

Linearity and spectral performance of the filter strongly dependent upon the linearity of the transconductor

Often can not justify elegant linearization strategies in the transconductors because of speed, area, and power penalties

Seminal Work on the OTA

RCA

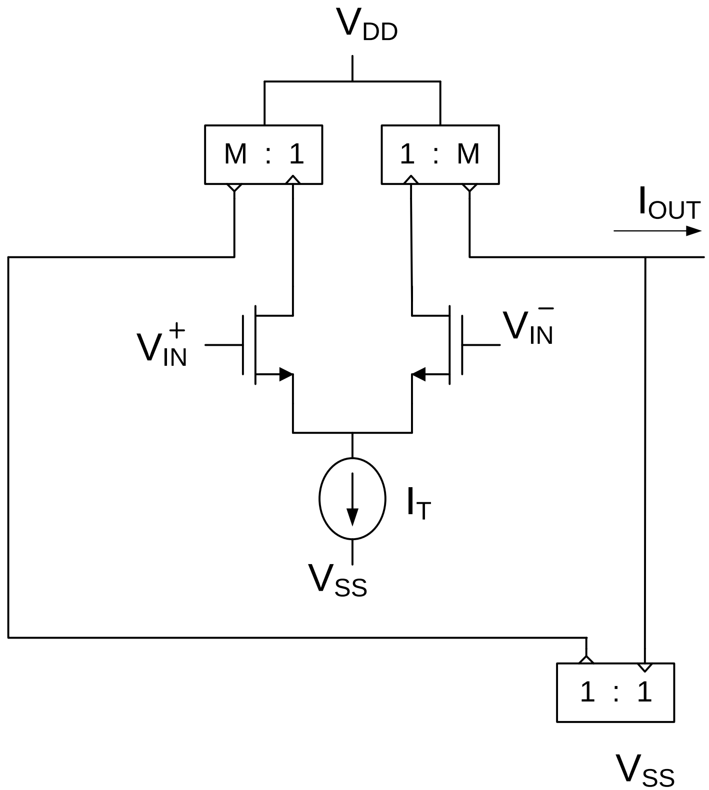
OTA Obsoletes Op Amp

by C.F. Wheatley
H.A. Wittlinger

From:

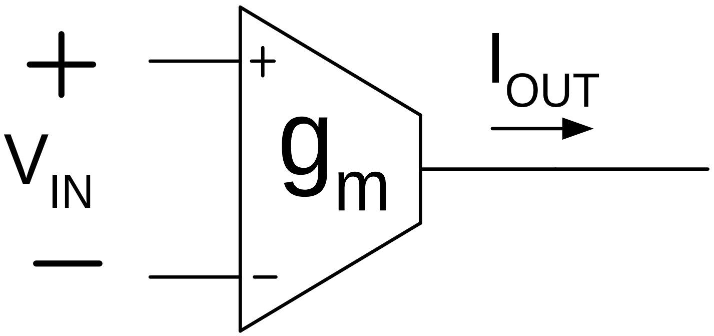
1969 N.E.C. PROCEEDINGS
December 1969

Current Mirror Op Amp W/O CMFB



$$g_{mEQ} = Mg_{m1}$$

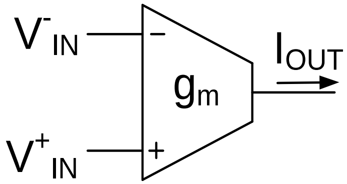
Often termed an OTA

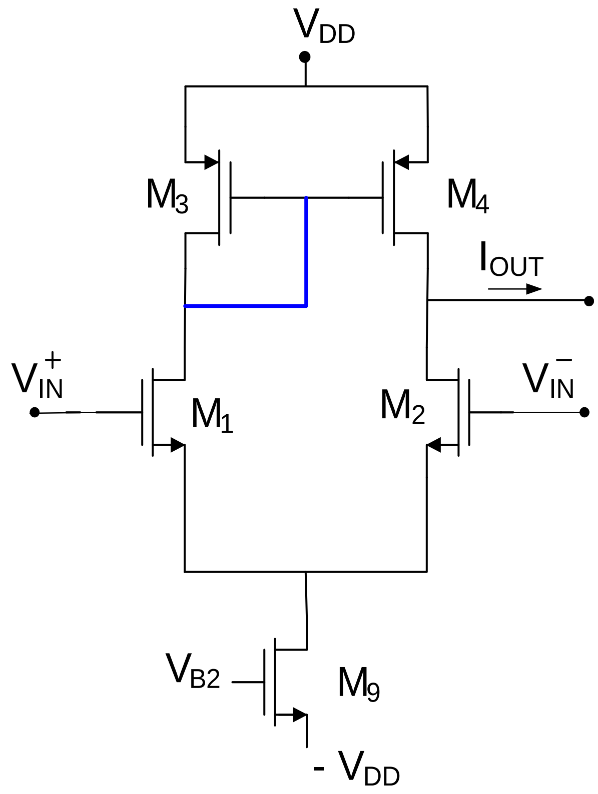


Introduced by Wheatley and Whitlinger in 1969

$$I_{OUT} = g_m V_{IN}$$

Basic OTA based upon differential pair

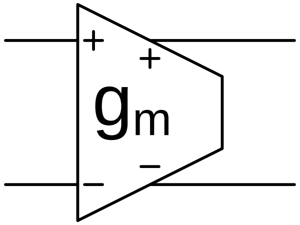


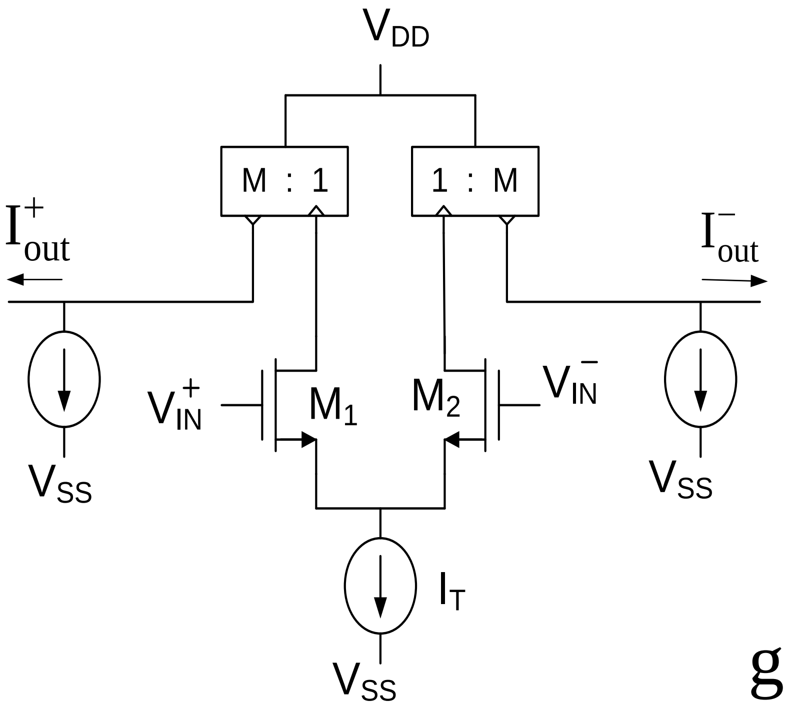


$$g_m = g_{m1}$$

Assume M_1 and M_2 matched,
M_3 and M_4 matched

Differential output OTA based upon differential pair

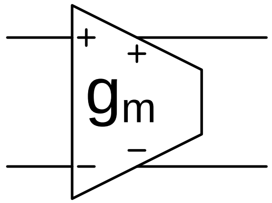


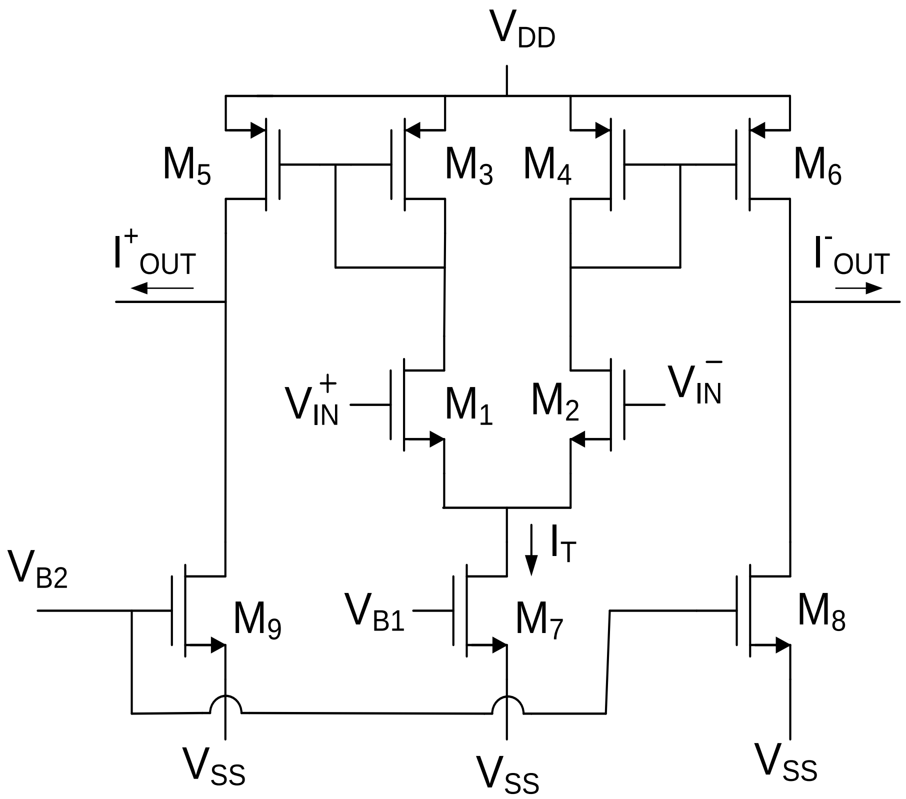


$$g_m = \frac{g_{m1}}{2} M$$

CMFB needed for the two output biasing current sources

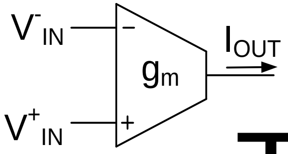
Differential output OTA based upon differential pair



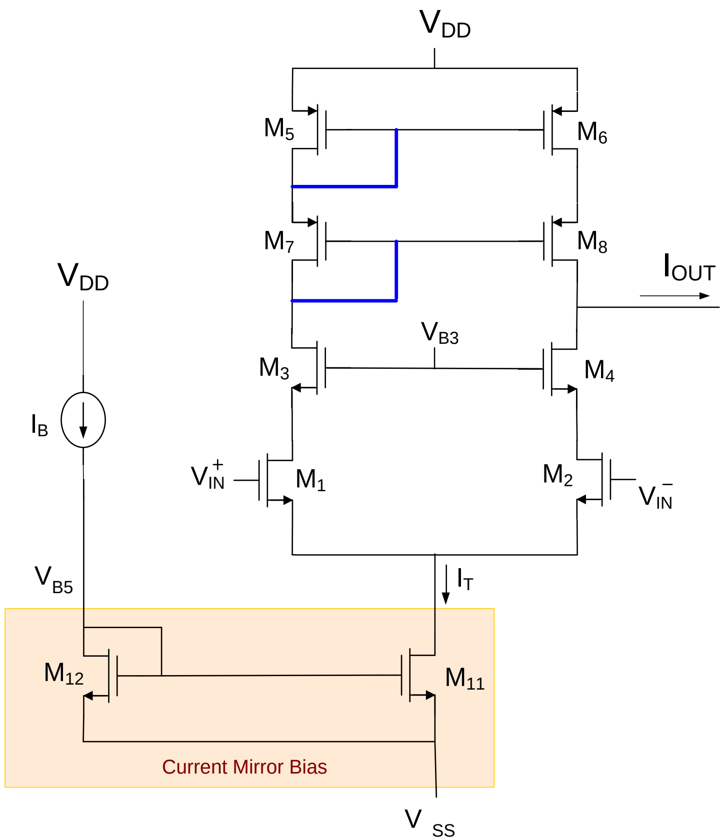


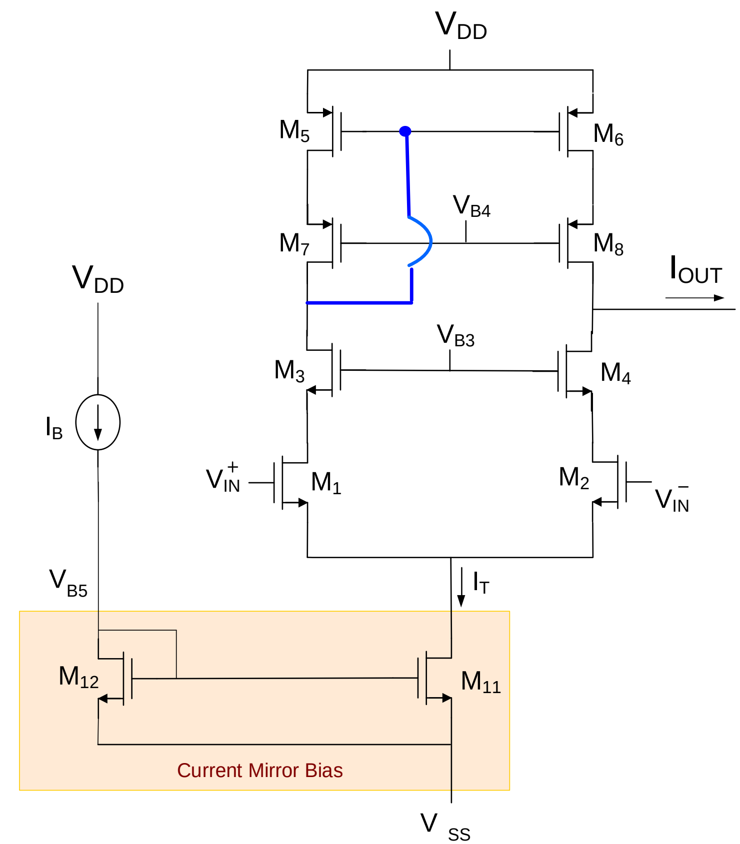
$$g_m = \frac{g_{m1}}{2} M$$

CMFB needed for the two output biasing current sources



Telescopic Cascode OTA



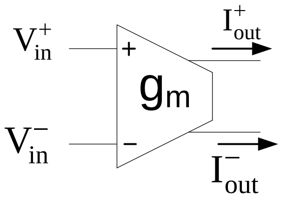


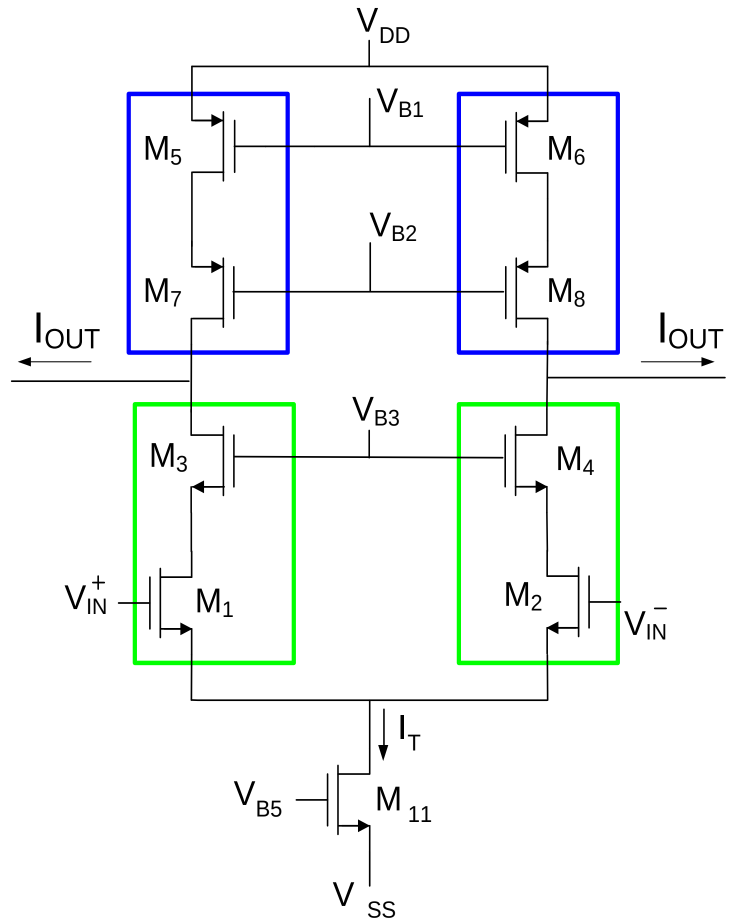
Standard p-channel Cascode Mirror

Wide-Swing p-channel Cascode Mirror

- Current-Mirror p-channel Bias to Eliminate CMFB
- Only single-ended output available

Telescopic Cascode OTA

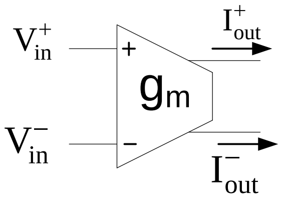


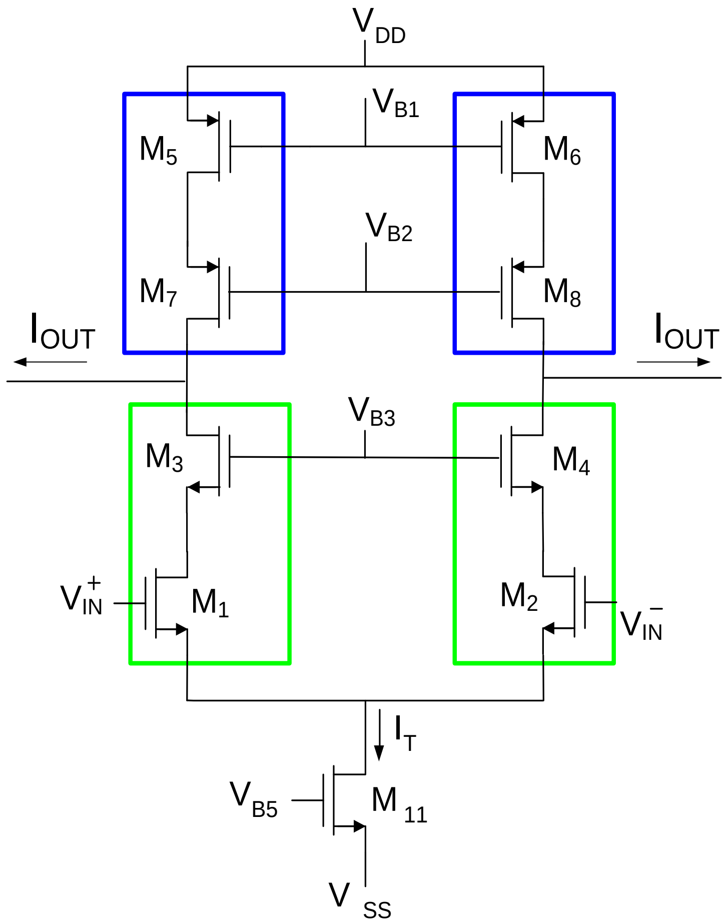


CMFB needed

Review from last lecture

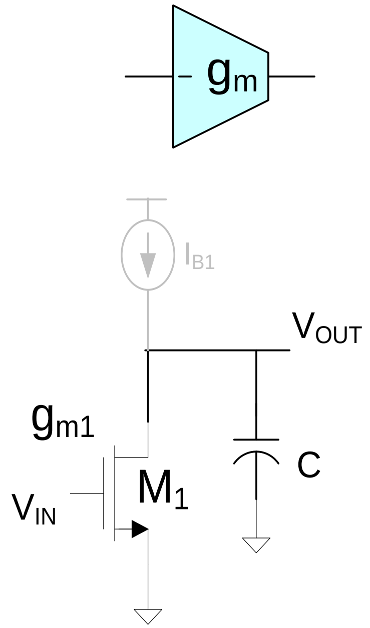
Telescopic Cascode OTA



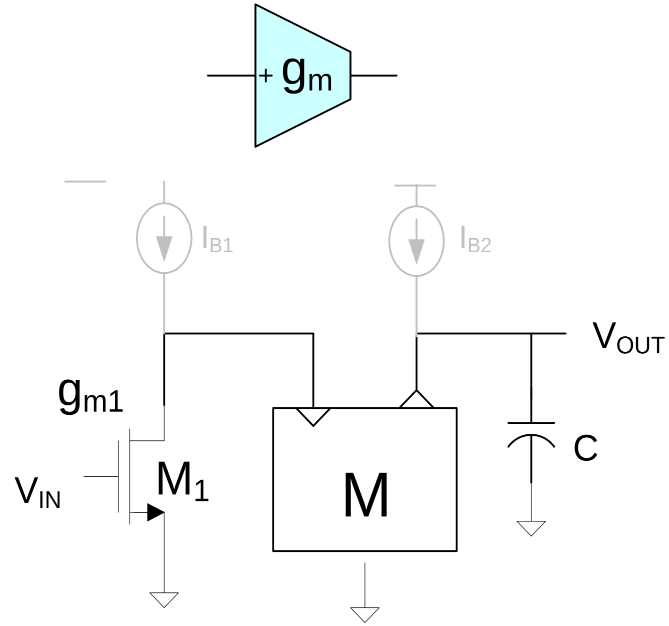


CMFB needed

Single-ended High-Frequency TA

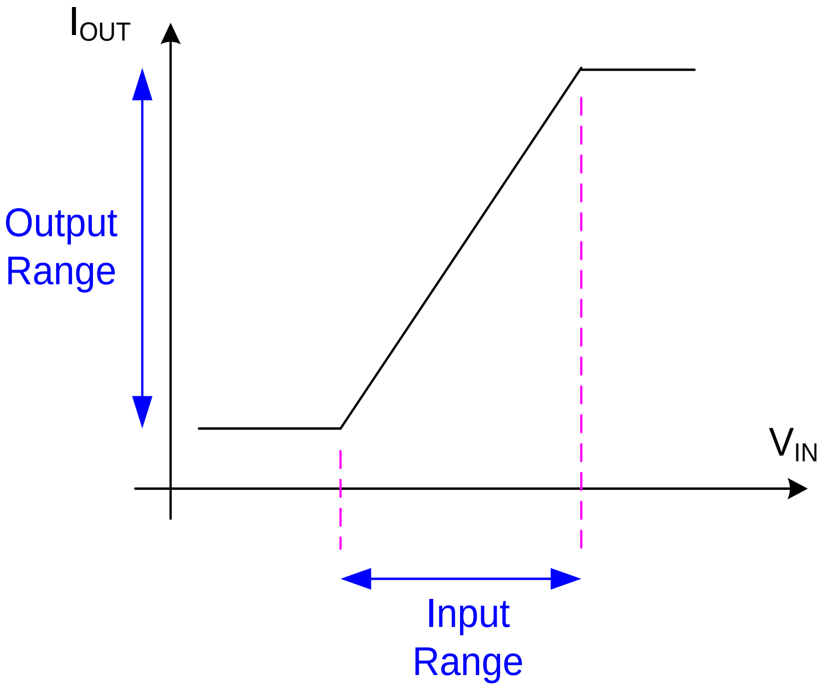


$$g_m = -g_{m1}$$



$$g_m = Mg_{m1}$$

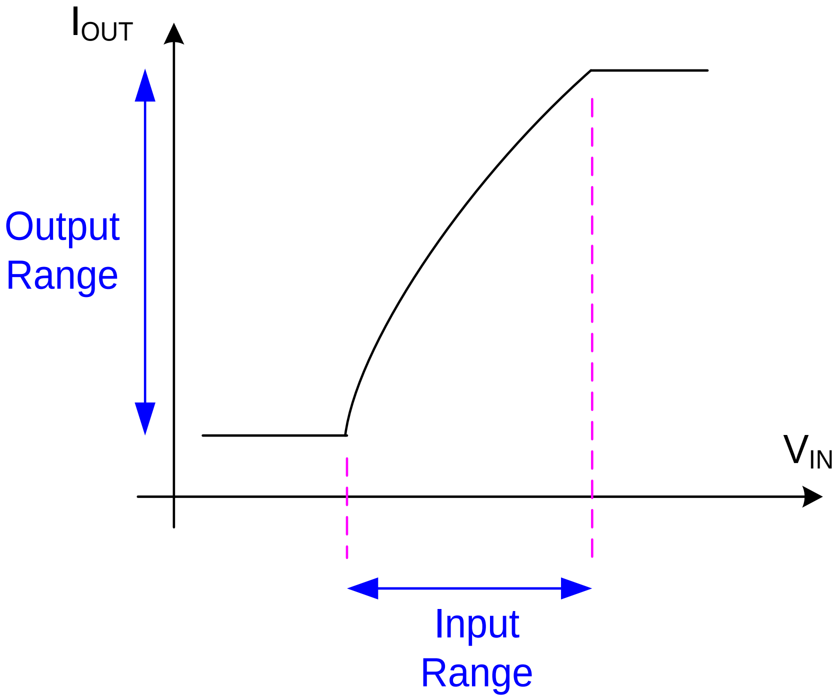
Signal Swing and Linearity



Ideal Scenario:

Completely Linear over Input and Output Range

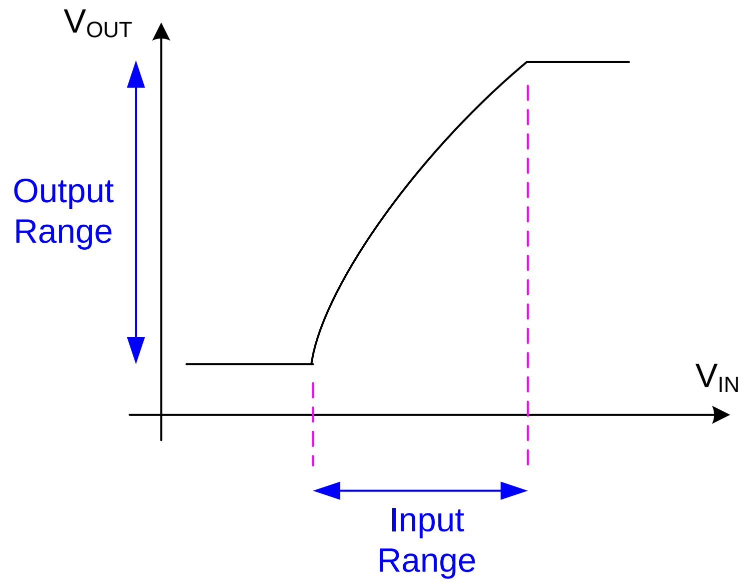
Signal Swing and Linearity

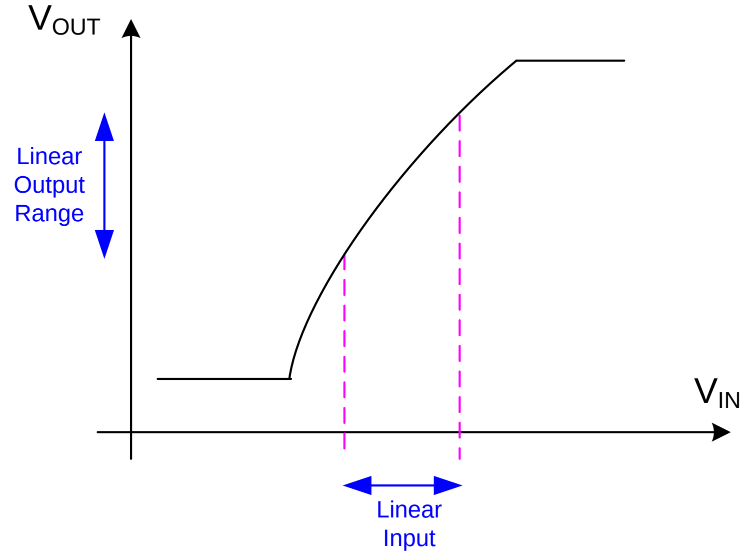


Realistic Scenario:

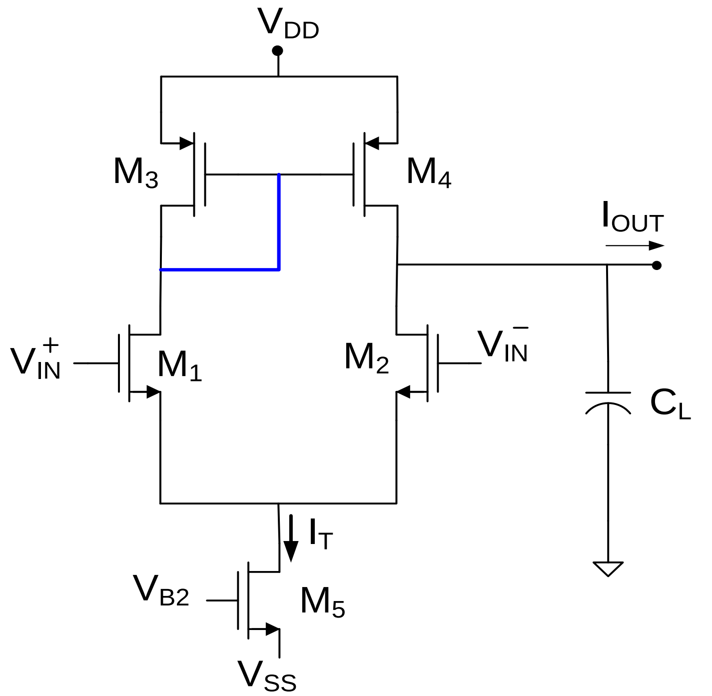
- Modest Nonlinearity throughout Input Range
- But operation will be quite linear over subset of this range

Signal Swing and Linearity



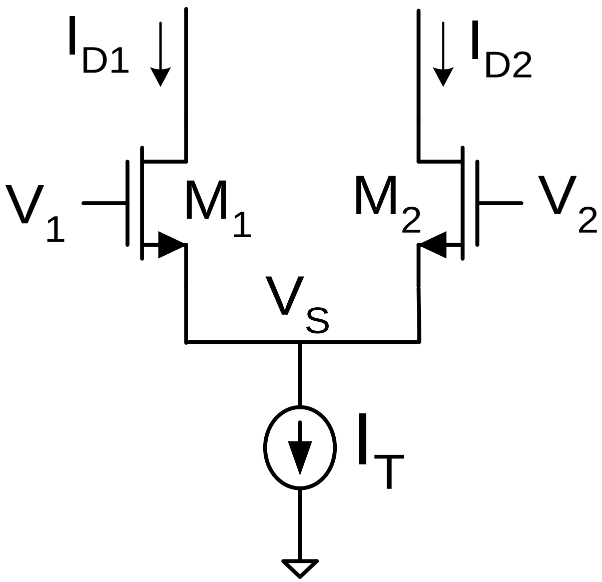


Linearity of Amplifiers

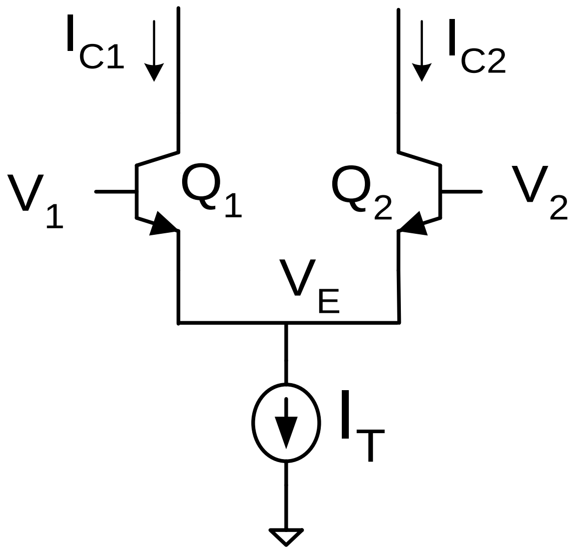


Strongly dependent upon linearity of transconductance of differential pair

Differential Input Pairs

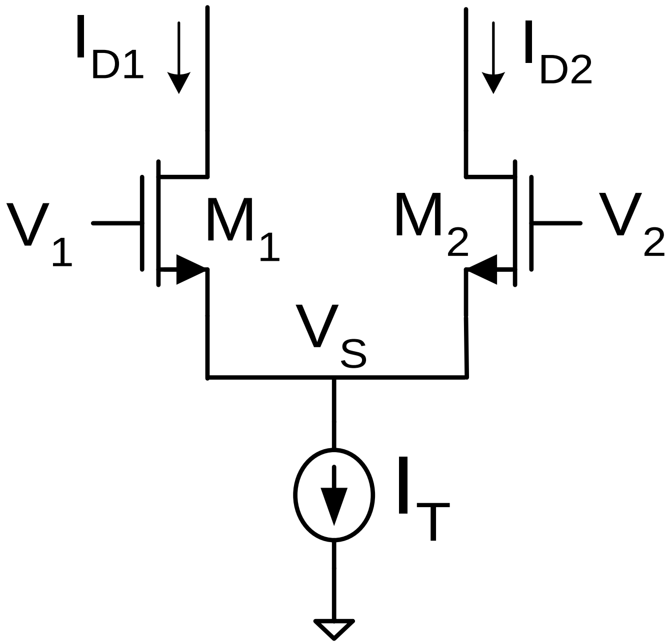


MOS Differential Pair



Bipolar Differential Pair

MOS Differential Pair



$$I_{D1} = \frac{\mu C_{OX} W}{2L}\left(V_1 - V_S - V_T\right)^2$$

$$I_{D2} = \frac{\mu C_{OX} W}{2L}\left(V_2 - V_S - V_T\right)^2$$

$$I_{D1} + I_{D2} = I_T$$

$$\sqrt{I_{D1}}\sqrt{\frac{2L}{\mu C_{OX} W}} = V_1 - V_S - V_T$$

$$\sqrt{I_{D2}}\sqrt{\frac{2L}{\mu C_{OX} W}} = V_2 - V_S - V_T$$

$$V_d = V_2 - V_1$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

MOS Differential Pair

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

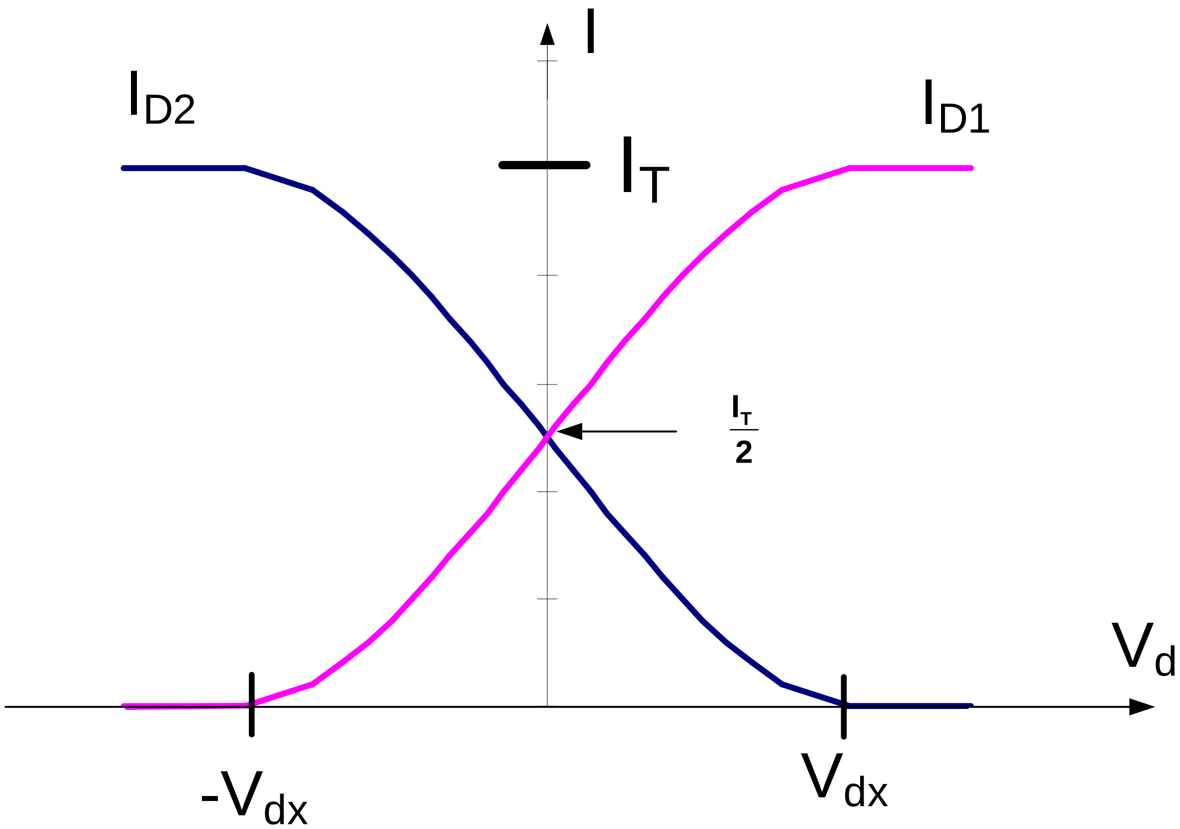
What values of V_d will cause all of the current to be steered to the left or the right ?

$$V_{dx} = \pm\sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T}\right)$$

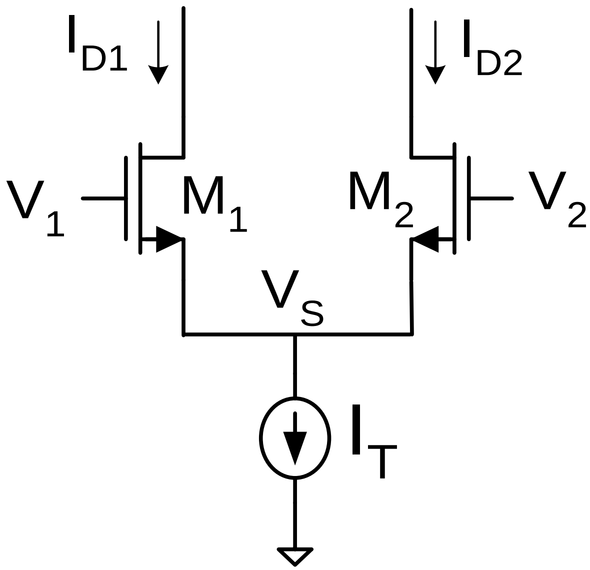
Transfer Characteristics of MOS Differential Pair

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

$$V_{dx} = \pm\sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T}\right)$$



Q-point Calculations for MOS Differential Pair



$$\frac{I_T}{2} = \frac{\mu C_{OX} W}{2L}\left(V_{EB}\right)^2$$

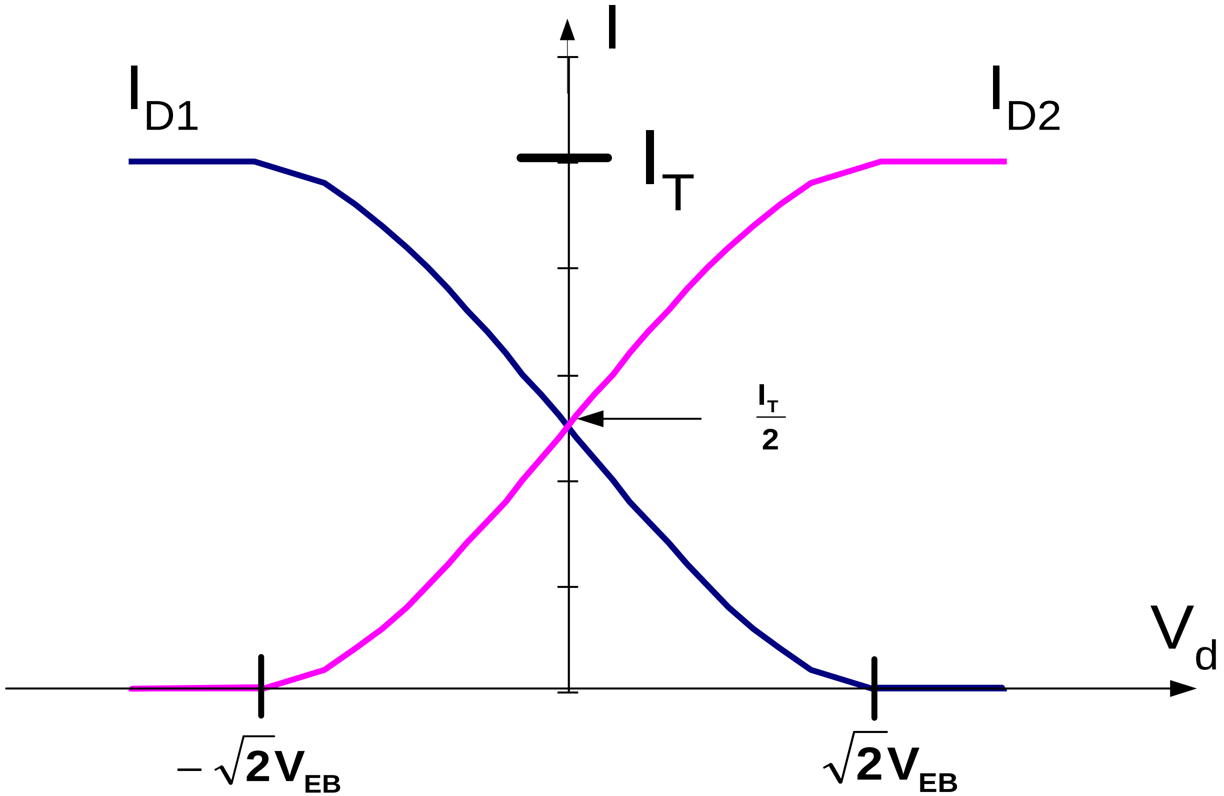
$$V_{EB} = \sqrt{I_T}\sqrt{\frac{L}{\mu C_{OX} W}}$$

Observe !!

$$V_{dx} = \pm\sqrt{2}V_{EB}$$

Transfer Characteristics of MOS Differential Pair

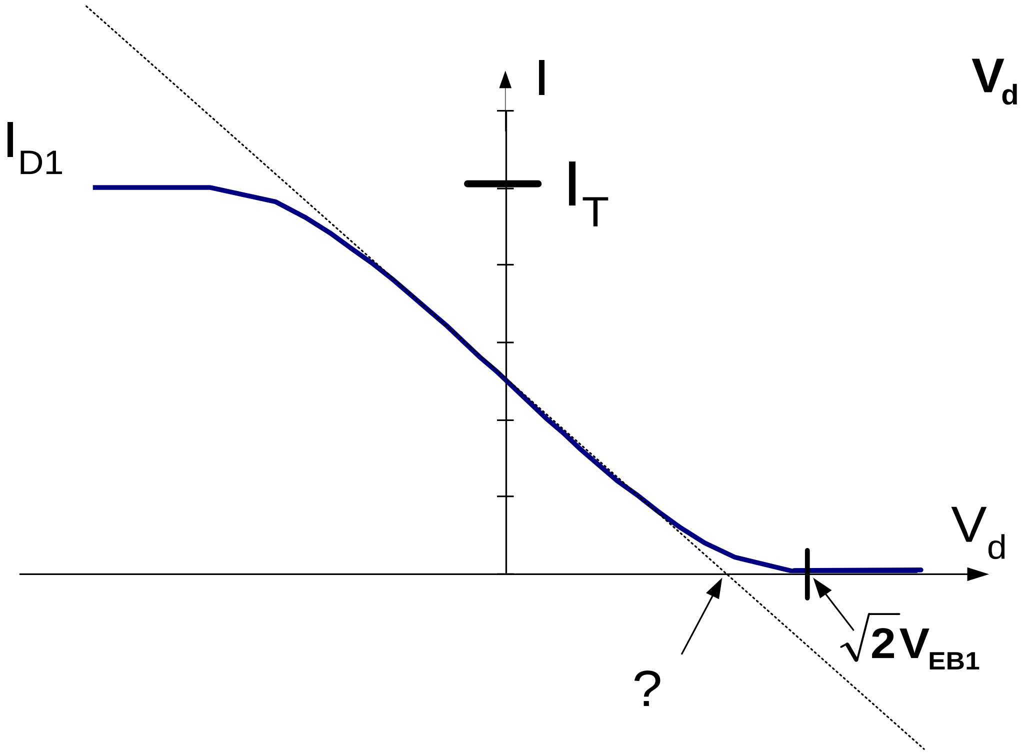
$$V_d = \sqrt{\frac{2L}{\mu C_{ox} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$



V_{EB} affects linearity

How linear is the amplifier ?

How linear is the amplifier ?



$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

Consider the fit line:

$$I = mV_d + h$$

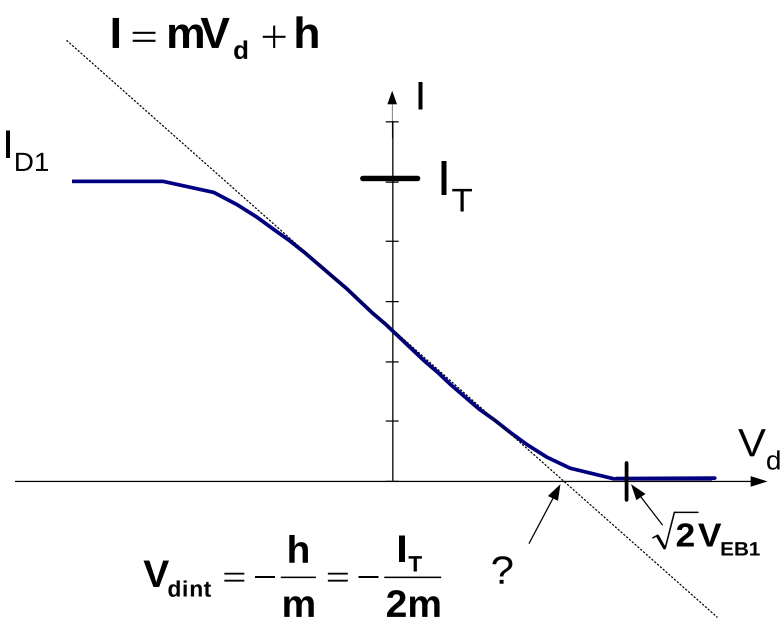
When V_d=0, I=I_T/2, thus

$$h = \frac{I_T}{2}$$

$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m}$$

$$m = \left.\frac{\partial I_{D1}}{\partial V_d}\right|_{Q-pt}$$

How linear is the amplifier ?



$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m}$$

$$m = \left.\frac{\partial I_{D1}}{\partial V_d}\right|_{Q-pt}$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

$$\frac{\partial V_d}{\partial I_{D1}} = \sqrt{\frac{2L}{\mu C_{OX} W}}\left.\left(\frac{1}{2}(I_T - I_{D1})^{-1/2}(-1) - \frac{1}{2}(I_{D1})^{-1/2}\right)\right|_{Q-point}$$

$$\frac{\partial V_d}{\partial I_{D1}} = -2\sqrt{\frac{L}{\mu C_{OX} W}}\sqrt{\frac{1}{I_T}}$$

$$\sqrt{\frac{L}{\mu C_{OX} W}} = \frac{V_{EB1}}{\sqrt{I_T}}$$

$$\frac{\partial V_d}{\partial I_{D1}} = -2\frac{V_{EB1}}{I_T}$$

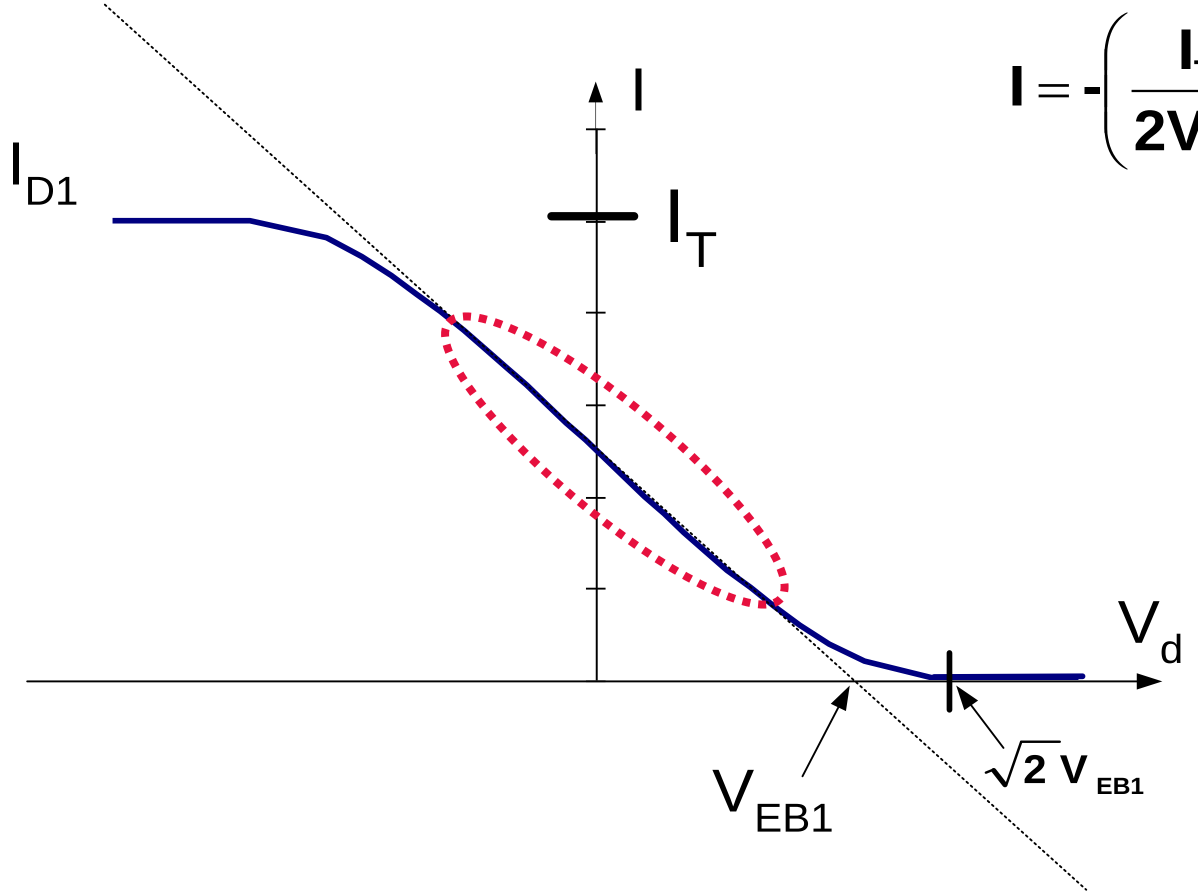
$$m = \left.\frac{\partial I_{D1}}{\partial V_d}\right|_{Q-pt} = -\frac{I_T}{2V_{EB1}}$$

$$V_{dint} = V_{EB1}$$

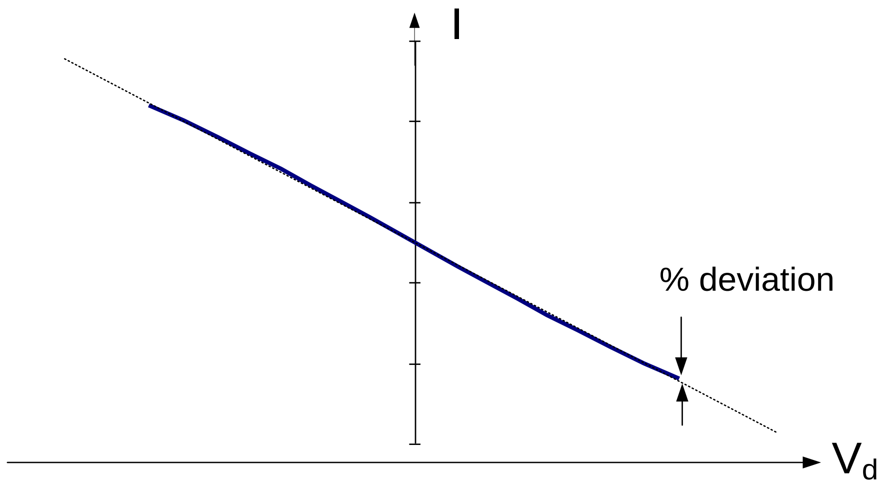
How linear is the amplifier ?

$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m} = V_{EB1}$$

$$I = -\left(\frac{I_T}{2V_{EB1}}\right)V_d + \frac{I_T}{2}$$



How linear is the amplifier ?

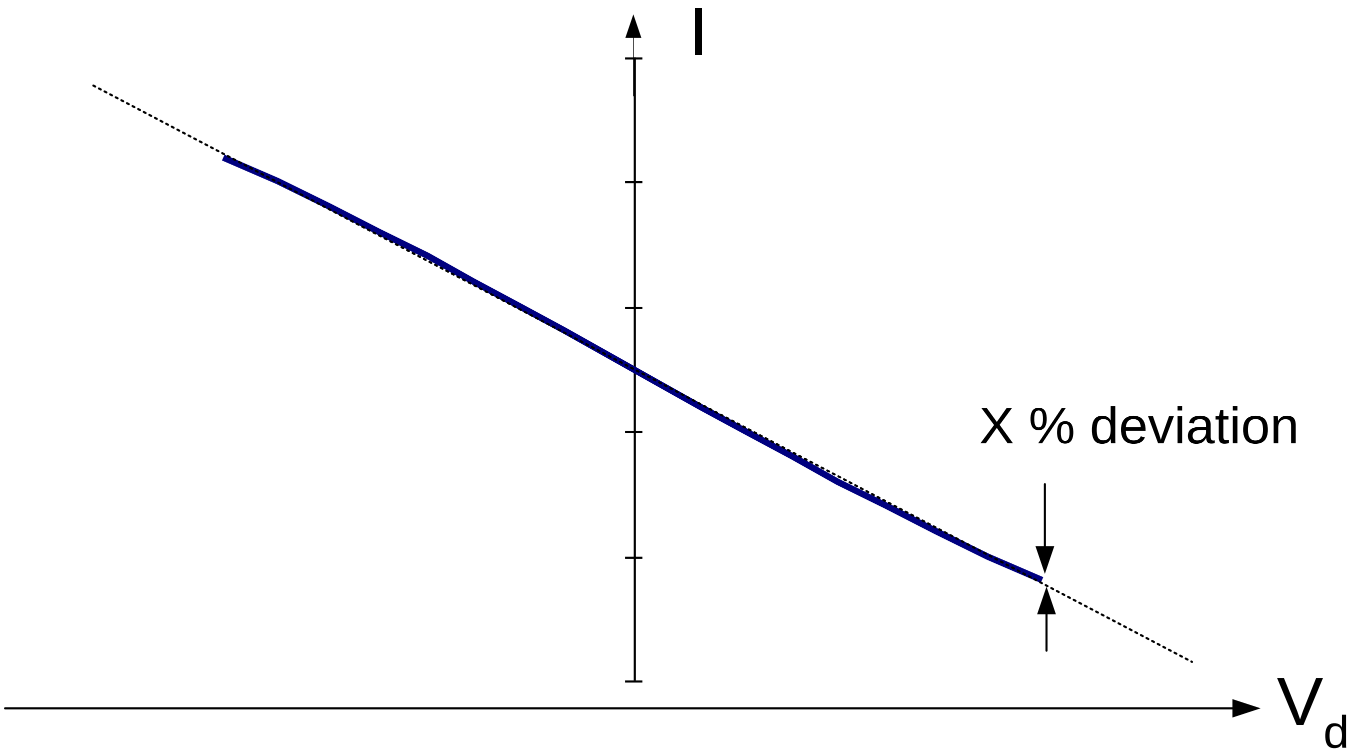


It can be shown that the deviation from the line in % is given by

$$\theta = 100\% \left(1 - \sqrt{1 - \frac{\left(V_d / V_{EB} \right)^2}{4}} \right)$$

Vd/VEB	θ	Vd/VEB	θ	Vd/VEB	θ
0.02	0.005	0.22	0.607	0.42	2.23
0.04	0.020	0.24	0.723	0.44	2.45
0.06	0.045	0.26	0.849	0.46	2.68
0.08	0.080	0.28	0.985	0.48	2.92
0.1	0.125	0.3	1.13	0.5	3.18
0.12	0.180	0.32	1.29	0.52	3.44
0.14	0.245	0.34	1.46	0.54	3.71
0.16	0.321	0.36	1.63	0.56	4.00
0.18	0.406	0.38	1.82	0.58	4.30
0.2	0.501	0.4	2.02	0.6	4.61

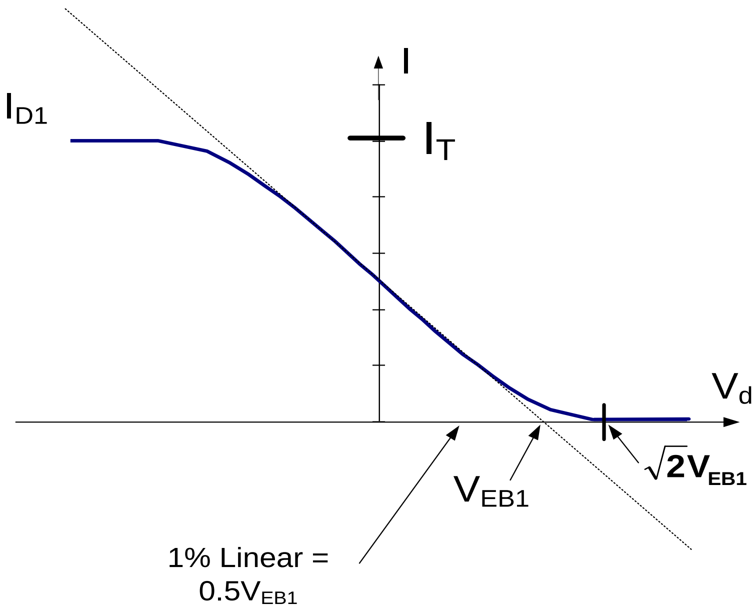
How linear is the amplifier ?



A 1% deviation from the straight line occurs at

$V_d \cong 0.3 V_{EB}$ and a 0.1% variation occurs at $V_d \cong \frac{V_{EB}}{10}$

What swings on drain currents are typical when using the differential pair in an amplifier?



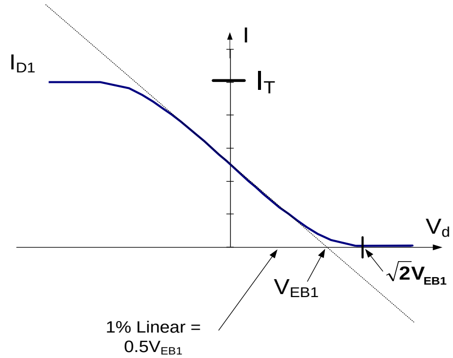
Assume the differential amplifier is the input stage to an op amp with gain Av and signal swing V_{OUTpp}

The differential swing at the input is thus

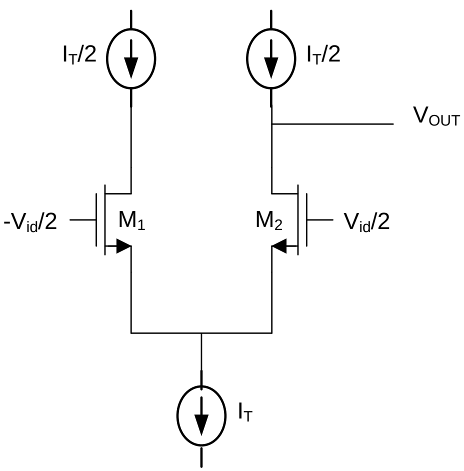
$$V_{INpp} = \frac{V_{OUTpp}}{A_V}$$

What swings on drain currents are typical when using the differential pair in an amplifier?

$$V_{INpp}=\frac{V_{OUTpp}}{A_V}$$



If the amplifier is the simple differential amplifier with current source loads



$$A_V = -\frac{g_{m1}}{2g_0} = \frac{2I_{DQ}/V_{EB1}}{2\lambda I_{DQ}}$$

$$A_V = -\frac{1}{\lambda V_{EB1}}$$

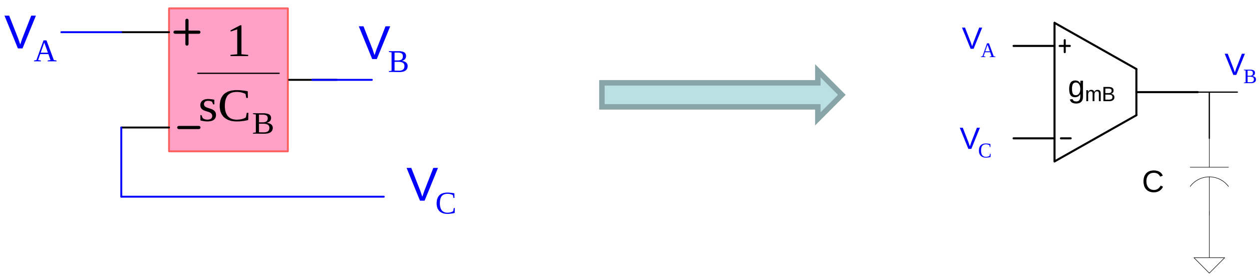
$$V_{INpp}=(\lambda V_{OUTpp})V_{EB1}$$

If $\lambda=.01V^{-1}$ and $V_{OUTpp}=5V$,

$$V_{INpp}=0.05V_{EB1}$$

This results in a very small nonlinearity and a very small change in current
When used in two-stage structure, even much smaller!

Programmable Filter Structures



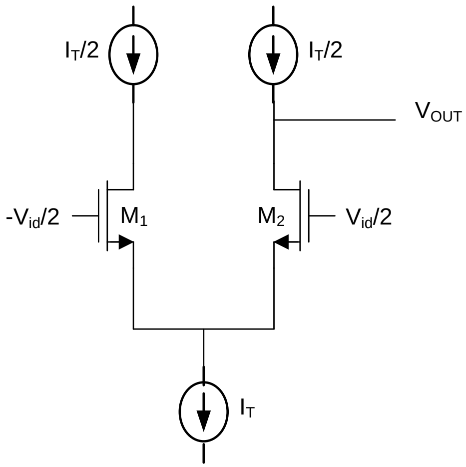
$$|\omega_0| = \frac{g_m}{C}$$

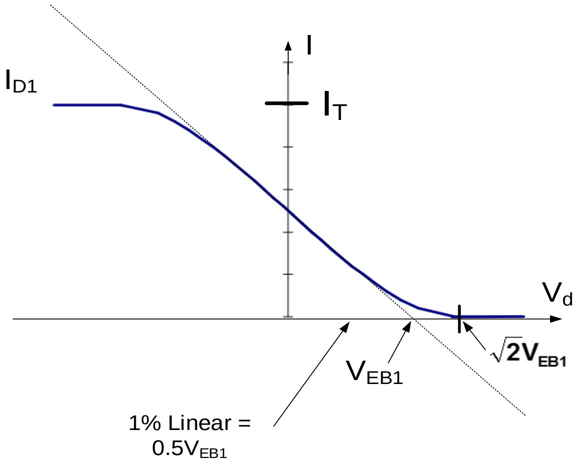
Often want to program or trim filters

Applicable in wide variety of filter architectures (here showing integrator-based)

Attractive to do this by adjusting g_m , in part, because g_m can be continuously adjustable with some transconductance devices

What input range is possible when using the tail current to program the OTA (i.e. after W/L fixed)?



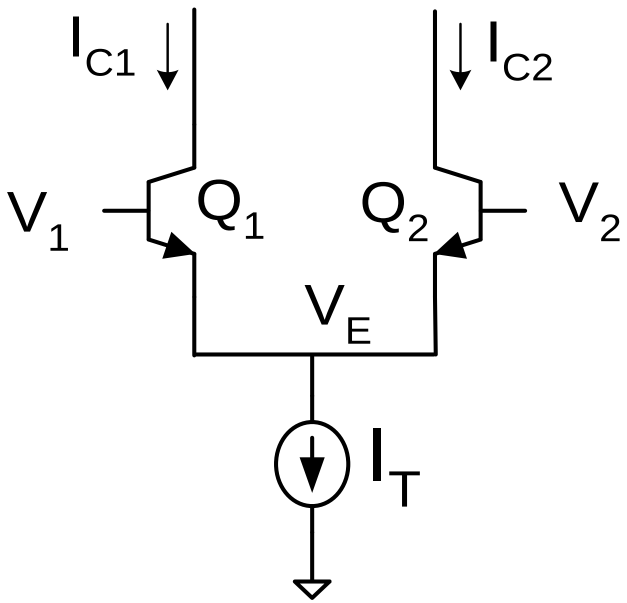


$$g_m = \mu C_{OX} \frac{W}{L} V_{EB} == \sqrt{I_T} \sqrt{\mu C_{OX} \frac{W}{L}}$$

$$\mathbf{V_{dx}} = \pm \sqrt{\frac{\mathbf{2L}}{\mathbf{\mu C_{ox} W}}} \left(\sqrt{\mathbf{I_T}} \right)$$

- Input signal swing decreases linearly with decreases in g_m for fixed W/L
- One decade reduction in g_m results in one decade decrease in signal swing
- One decade reduction in g_m requires two decade decrease in I_T
- Though MOS OTA can have very good single swing with large V_{EB}, very limited tail current programmability with basic MOS OTA
- There are, however, other ways to program MOS OTA without big penalty in signal swing

Bipolar Differential Pair



$$\left.\begin{aligned} I_{C1} &= J_S A_{E1} e^{\frac{V_1 - V_E}{V_t}} \\ I_{C2} &= J_S A_{E2} e^{\frac{V_2 - V_E}{V_t}} \\ I_{C1} + I_{C2} &= I_T \end{aligned}\right\}$$

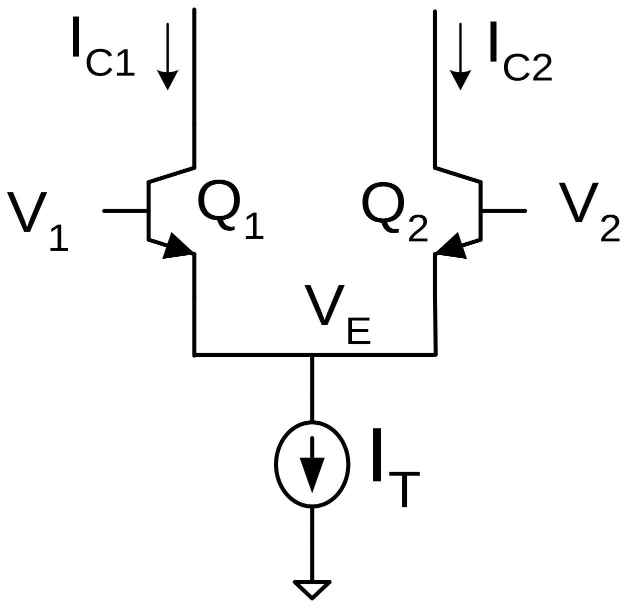
$$V_1 = V_E + V_t \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)$$

$$V_2 = V_E + V_t \ln\left(\frac{I_{C2}}{J_S A_{E2}}\right)$$

$$V_d = V_2 - V_1$$

$$V_d = V_t\left(\ln\left(\frac{I_{C2}}{J_S A_{E2}}\right) - \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)\right) \overset{A_{E1}=A_{E2}}{=} V_t \ln\left(\frac{I_{C2}}{I_{C1}}\right)$$

Bipolar Differential Pair



$$\mathbf{V_d = V_2 - V_1}$$

$$\mathbf{V_d = V_t\left(\ln\left(\frac{I_{C2}}{J_S A_{E2}}\right) - \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)\right) \overset{A_{E1}=A_{E2}}{=} V_t \ln\left(\frac{I_{C2}}{I_{C1}}\right)}$$

$$\mathbf{V_d = V_t \ln\left(\frac{I_T - I_{C1}}{I_{C1}}\right)}$$

$$\mathbf{V_d = V_t \ln\left(\frac{I_{C2}}{I_T - I_{C2}}\right)}$$

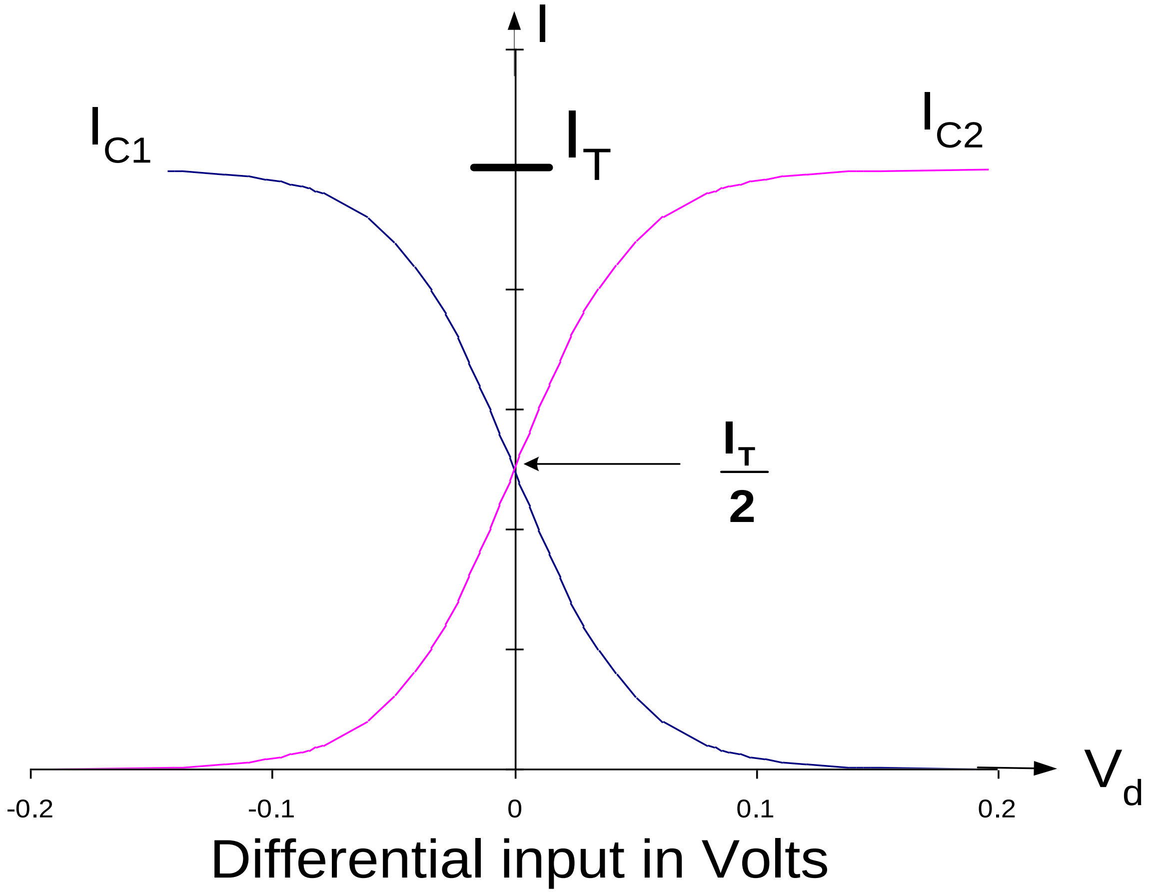
At $I_{C1}=I_{C2}=I_T/2$, $V_d=0$

As I_{C1} approaches 0, V_d approaches infinity

As I_{C1} approaches I_T, V_d approaches minus infinity

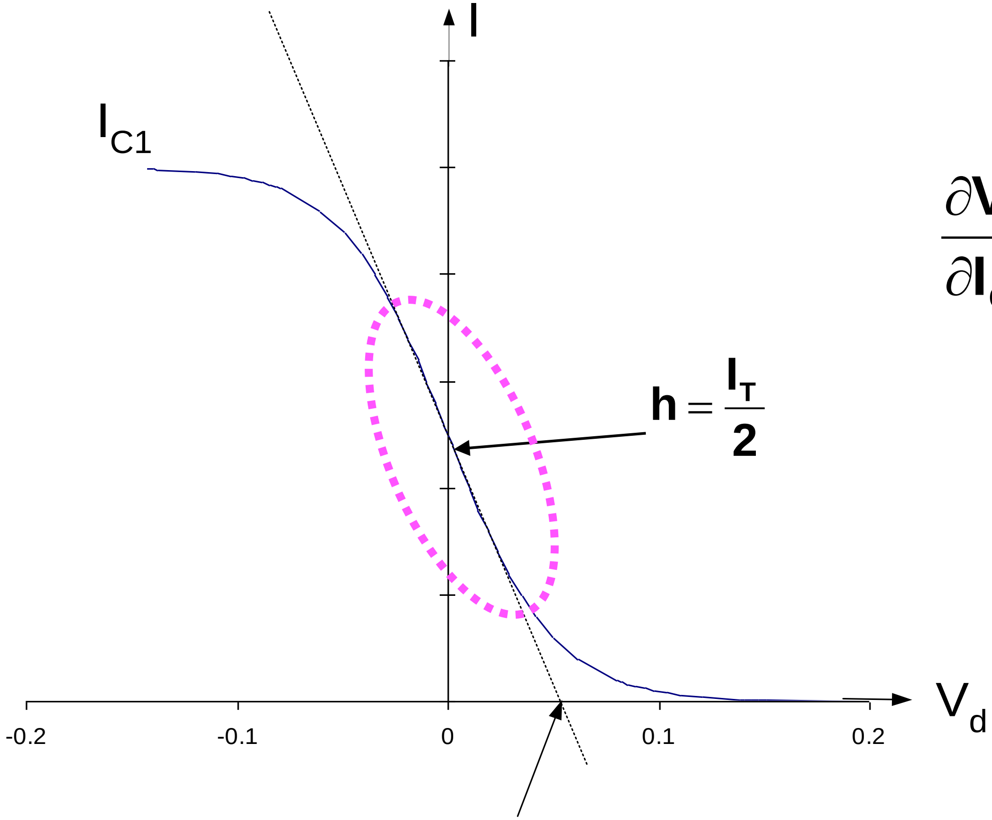
Transition much steeper than for MOS case

Transfer Characteristics of Bipolar Differential Pair



Transition much steeper than for MOS case
Asymptotic Convergence to 0 and I_T

Signal Swing and Linearity of Bipolar Differential Pair



$$I_{FIT} = mV_d + h$$

$$h = \frac{I_T}{2}$$

$$V_{dint} = -\frac{h}{m} = ?$$

$$m = \left.\frac{\partial I_{C1}}{\partial V_d}\right|_{Q-point}$$

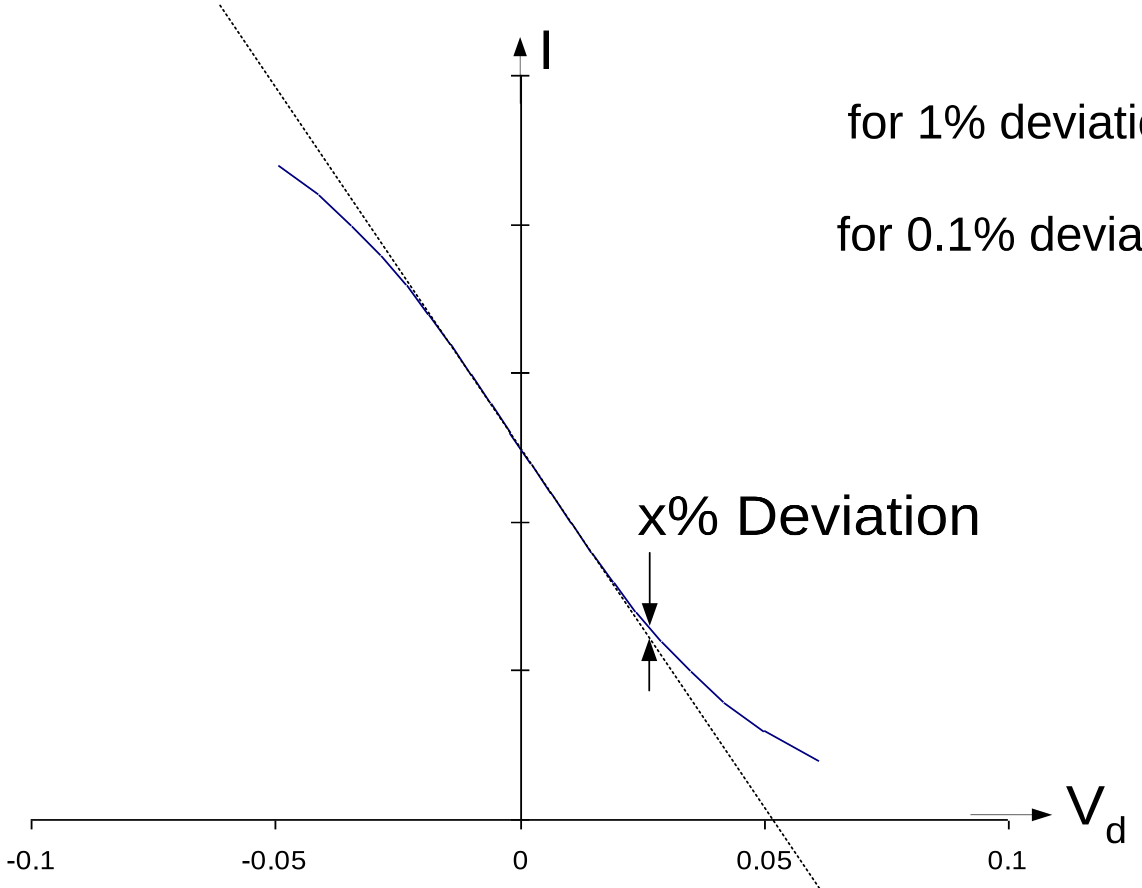
$$\left.\frac{\partial V_d}{\partial I_{C1}}\right|_{Q-point} = -V_t \left.\frac{I_T}{I_{C1}(I_T - I_{C1})}\right|_{I_{C1}=\frac{I_T}{2}}$$

$$\left.\frac{\partial V_d}{\partial I_{C1}}\right|_{Q-point} = -\frac{4V_t}{I_T}$$

$$I_{FIT} = -\frac{I_T}{4V_t}V_d + \frac{I_T}{2}$$

$$V_{dint} = -\frac{h}{m} = 2V_t$$

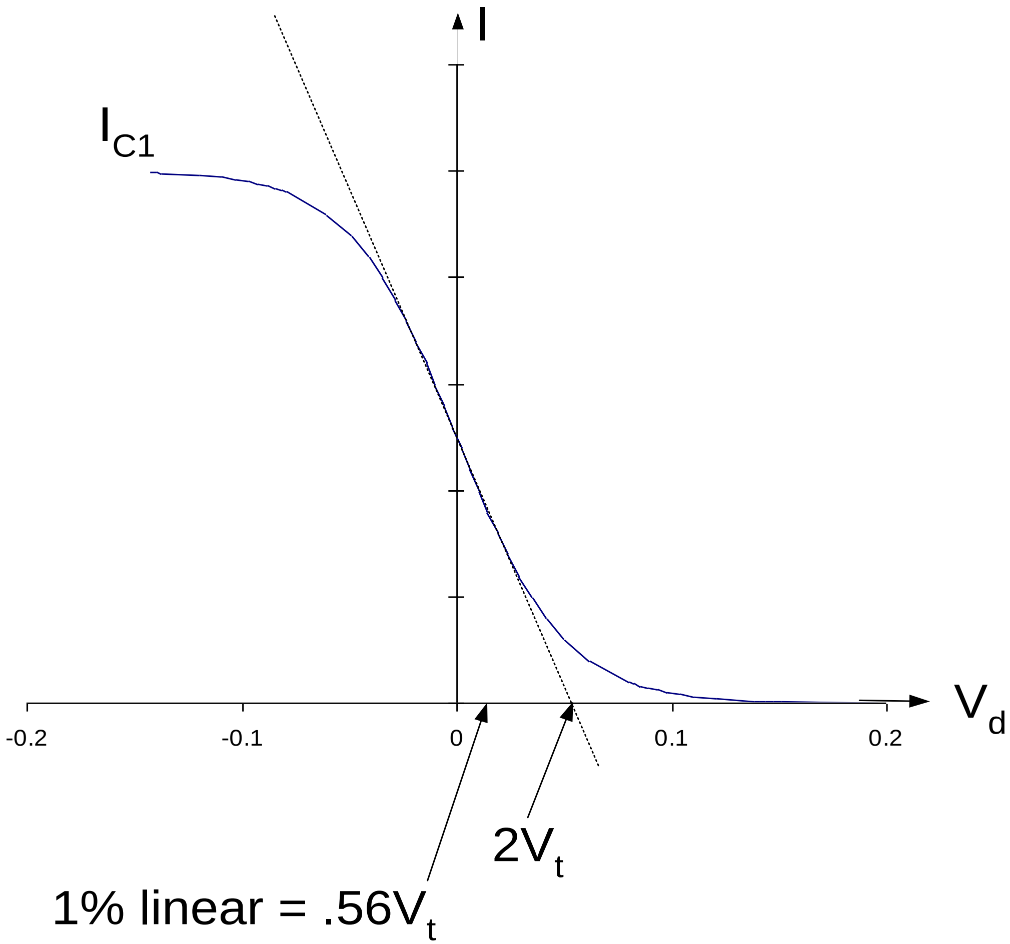
Signal Swing and Linearity of Bipolar Differential Pair



for 1% deviation, V_d=.56V_t

for 0.1% deviation, V_d=.27V_t

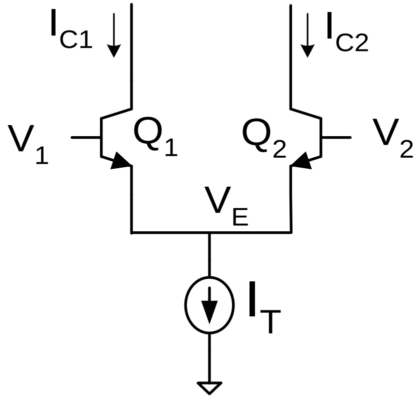
Signal Swing and Linearity of Bipolar Differential Pair



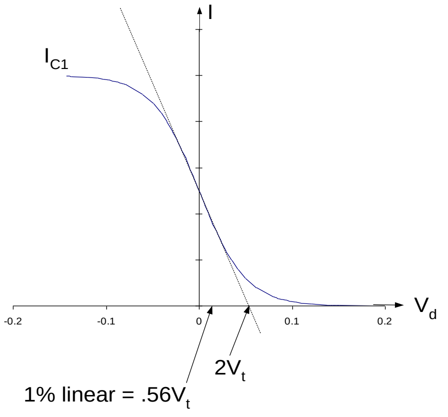
Signal Swing and Linearity Summary

- Signal swing of MOSFET can be rather large if V_{EB} is large but this limits gain
- Signal swing of MOSFET degrades significantly if V_{EB} is changed for fixed W/L
- Bipolar swing is very small but independent of g_m
- Multiple-decade adjustment of bipolar g_m is practical
- Even though bipolar input swing is small, since gain is often very large, this small swing does usually not limit performance in feedback applications

What input range is possible when using the tail current to program the OTA ?



$$g_m = \frac{I_T}{2V_t}$$



- Input signal swing not affected by I_T
- Multi-decade adjustment of g_m with I_T without degrading signal swing

Stay Safe and Stay Healthy !

End of Lecture 31